Biostats 101

Seminar 4: t-Tests and $\chi^2$ Tests

Introduction

Outline
- Research Process
  - Point Estimates
  - Confidence Intervals
  - Hypothesis Testing
- t-Tests
  - Overview
  - Procedure
  - Example
- $\chi^2$ Tests
  - Overview
  - Procedure
  - Example
- Conclusions

Statistics in the Research Process

Outline
- Research Process
  - Point Estimates
  - Confidence Intervals
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- t-Tests
  - Overview
  - Procedure
  - Example
- $\chi^2$ Tests
  - Overview
  - Procedure
  - Example
- Conclusions
t-Test

### Types of t-Tests

- **1-sample t-test**
  \[ H_0: \mu = \mu_0 \]
  \[ H_1: \mu \neq \mu_0 \]
  \[ t = \frac{\bar{X} - \mu_0}{s} \frac{1}{\sqrt{n}} \]

- **2-sample (independent) t-test**
  \[ H_0: \mu_1 = \mu_2 \]
  \[ H_1: \mu_1 \neq \mu_2 \]
  \[ t = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1, \bar{X}_2} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \]

- **Paired t-test**
  \[ H_0: \Delta \mu = 0 \]
  \[ H_1: \Delta \mu \neq 0 \]
  \[ t = \frac{\bar{X}_{\Delta}}{s_{\bar{X}_{\Delta}}} \frac{1}{\sqrt{n}} \]

- **2-sample paired t-test**
  \[ H_0: \Delta \mu = 0 \]
  \[ H_1: \Delta \mu \neq 0 \]
  \[ t = \frac{\bar{X}_{\Delta}}{s_{\bar{X}_{\Delta}}} \frac{1}{\sqrt{n}} \]

### Example (Rosner, 2006)
- Population
  - 35-39 year old premenopausal women
- Intervention/Predictor
  - Use of oral contraceptives
    - \( n_{OC} = 8 \)
    - \( n_{non-OC} = 21 \)
- Outcome
  - Systolic blood pressure

### t-Test Procedure

- **Null and Alternative Hypotheses**
  \[ H_0: \mu_1 = \mu_2 \]
  \[ H_1: \mu_1 \neq \mu_2 \]

- **Significance Level**
  \( \alpha = 0.05 \)

- **Test Statistic**
  \( t = \) varies

- **Critical value or p-value**
  - p-value: probability of observing t (or something more extreme) given \( H_0 \)
  - Decide whether the observed t is inconsistent with \( H_0 \)
  - Reject/fail to reject \( H_0 \)
t-Test

Types of t-Tests

- 1-sample t-test
  \[ t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \]  
- 2-sample (independent) t-test
  \[ t = \frac{\overline{\Delta} \pm t_{\alpha, df} \cdot s_{\Delta}}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \]  
- Paired t-test
  \[ t = \frac{\overline{\Delta} \pm t_{\alpha, df} \cdot s_{\Delta} / \sqrt{n}}{s_{\Delta} / \sqrt{n}} \]  
- 2-sample paired t-test
  \[ t = \frac{\overline{\Delta} \pm t_{\alpha, df} \cdot s_{\Delta} / \sqrt{n}}{s_{\Delta} / \sqrt{n}} \]

Hypotheses

- \( H_0: \mu_1 = \mu_2 \) or \( H_0: \mu_1 = \mu_2 = 0 \)
- \( H_1: \mu_1 \neq \mu_2 \) or \( H_1: \mu_1 \neq \mu_2 = 0 \)

Critical value

\[ t_{\alpha, df} \]  

Decisions

- Reject \( H_0 \) if \( |t| > t_{\alpha, df} \)  
- Fail to reject \( H_0 \) if \( |t| < t_{\alpha, df} \)

Test Statistic

\[ t = \frac{\overline{\Delta} \pm t_{\alpha, df} \cdot s_{\Delta}}{s_{\Delta} / \sqrt{n}} \]

Significance

\[ p = P(\text{obtained} \ t \mid \text{null hypothesis is true}) \]

Example

Null and Alternative Hypotheses

\[ H_0: \mu_1 \leq \mu_2 \]  
\[ H_1: \mu_1 > \mu_2 \]

Significance Level

\( \alpha = 0.05 \)

Test Statistic

\[ t = 0.74 \]

Critical value or p-value

\( p = 0.466 \)  
\( t = 0.74 < t_{0.05, 27} = 1.71 \)

Decision

- Reject \( H_0 \) if \( p < \alpha \)  
- Fail to reject \( H_0 \) if \( p \geq \alpha \)

Statistical Intervals

\[ \overline{\Delta} \pm t_{\alpha, df} \cdot s_{\Delta} / \sqrt{n} \]

Point Estimate

\[ \mu \]

Excel

\[ =T.DIST.2T(0.74,27) = 0.466 = p \]  
\[ =T.DIST.2T(0.74,27) \]

Conclusion

- The observed \( t \) value is consistent with \( H_0 \)

Test Results

- \( X_{OC} = 132.86 \) mm Hg  
- \( S_{OC} = 15.34 \) mm Hg  
- \( X_{HOC-OC} = 127.44 \) mm Hg  
- \( S_{HOC-OC} = 18.23 \) mm Hg

Hypothesis

\[ H_0: \mu_{HOC-OC} = 0 \]  
\[ H_1: \mu_{HOC-OC} > 0 \]

Research

- \( \mu \)  
- \( \sigma \)  
- \( n \)

Summary

- \( \bar{x} \pm \hat{\sigma} \cdot \sqrt{n} \)
- \( X \pm S \)
- \( \mu \pm \sigma \cdot \sqrt{n} \)
- \( \overline{\Delta} \pm \hat{s}_{\Delta} / \sqrt{n} \)

Graphics

- Box plot
- Normal distribution
- Confidence interval

2/25/2014
**χ² Test**

**Observed and Expected Tables**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Disease</th>
<th>No Disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>d</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected</th>
<th>Disease</th>
<th>No Disease</th>
</tr>
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<tbody>
<tr>
<td>Control</td>
<td>a+c</td>
<td>b+d</td>
</tr>
<tr>
<td></td>
<td>(x+y)(x+z)</td>
<td>(x+y)(x+z)</td>
</tr>
</tbody>
</table>

**H0 Testing Procedure**

- Null and Alternative Hypotheses
  - H0: p₁ = p₂
  - H₁: p₁ ≠ p₂
- Significance Level α = 0.05
- Test Statistic
  - χ² = \sum \frac{(O-E)^2}{E}
- Critical value or p-value
  - p-value: probability of observing χ² (or something more extreme) given H₀
- Decide whether the observed χ² is inconsistent with H₀
  - Reject/fail to reject H₀

**Example (Fowler, 2006)**

- Population: *Staphylococcus aureus* infection
- Intervention: Daptomycin (treatment), or Standard therapy (control)
- Outcome: Treatment success 42 days after treatment

**Critical/ p-value**

**Decision**

**χ² Test Procedure**

- Null and Alternative Hypotheses
  - H₀: p₁ = p₂
  - H₁: p₁ ≠ p₂
- Significance Level α = 0.05
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  - χ² = \sum \frac{(O-E)^2}{E}
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χ² Test

χ² Test Procedure
- Null and Alternative Hypotheses
  H₀: p₁=p₂, p₃
  H₁: p₁≠p₂, p₃
- Significance Level
  α=0.05
- Test Statistic
  \( \chi^2 = \sum (o_i - e_i)^2 / e_i \)
- Critical value or p-value
  - Critical value or p-value
  - Decide whether the observed χ² is inconsistent with H₀

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</tr>
<tr>
<td>48</td>
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χ² Test

χ² Test Procedure
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**χ² Test**

**χ² Test Procedure**
- **Null and Alternative Hypotheses**
  \[ H_0: p_1 = p_2 \]
  \[ H_1: p_1 \neq p_2 \]
- **Significance Level**
  \[ \alpha = 0.05 \]
- **Test Statistic**
  \[ \chi^2 = 0.296 \]
- **Critical value or p-value**
  \[ p = 0.586 \]
- **Decision**
  \[ p \text{-value} > \alpha \rightarrow 0.586 > 0.05 \]
  Cannot reject \( H_0 \)

**Conclusions**
- t-test used when
  1. One or two group/category predictor
- Continuous, normally distributed outcome
- Tests means
- χ² test used when
- Predictor is categorical
- Outcome is categorical
- Tests proportions
- Same procedure for both tests
- Hypotheses change
- Test statistic changes (formula and distribution)

### Biostats 101

<table>
<thead>
<tr>
<th>Topic</th>
<th>Speaker</th>
<th>Time</th>
<th>Location</th>
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<tr>
<td>1</td>
<td>Testing and Chi-square Tests</td>
<td>Bruce Oakley, MD</td>
<td>1/28/14</td>
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<tr>
<td>2</td>
<td>T-tests and Chi-square Tests</td>
<td>Sam McNair, PhD</td>
<td>2/25/14</td>
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<td>3</td>
<td>Power and Sample Size</td>
<td>Sam McNair, PhD</td>
<td>3/25/14</td>
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<td>Basics of Linear Regression</td>
<td>Bruce Oakley, MD</td>
<td>4/29/14</td>
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<td>6</td>
<td>Basics of Meta-Analysis</td>
<td>Sam McNair, PhD</td>
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For statistical help, go to: [http://cctsi.ucdenver.edu/Research-Resources/Pages/Biostats101-Research-Design.aspx](http://cctsi.ucdenver.edu/Research-Resources/Pages/Biostats101-Research-Design.aspx)

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