Magnetic Field Penetration Into a Metal Enclosure Using an ELF/VLF Loop Antenna

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Abstract—The ability of extremely and very low frequency (ELF/VLF, 0–30 kHz) radio waves to penetrate conductive media is well established. Magnetic field penetration into a thin but highly conductive box using an ELF/VLF loop antenna transmitter is investigated. The work is relevant for electromagnetic shielding of ELF/VLF sensors, defect detection, inductive power transfer, and through container imaging. Analytical solutions are reviewed for related shielding problems, however, determining the penetration through a realistic shield geometry and finite sized near-field source requires a numerical approach. Surface integral equation (SIE) methods are well suited for shield modeling due to the low surface area to volume ratio of the shield. Method of moment techniques have been successfully applied to solving SIEs in the past, however, enforcing algorithm stability at low frequencies is known to require considerable effort. To alleviate the low-frequency concerns, a high-order locally corrected Nyström (LCN) scheme is utilized to solve an SIE based on an augmented Müller formulation of Maxwell's equations. To validate the LCN simulations, an experiment is conducted using a loop antenna inside a 1.2 m aluminum cube of 2.7 mm thickness with an external ELF/VLF loop transmitter. Experimental results are shown to match within 3 dB of the LCN code predictions.

Index Terms—Computational electromagnetics, ELF/VLF shielding, extremely low frequency, loop antenna, near-field, nyström method, surface integral equation, very low frequency.

I. INTRODUCTION

ELF/VLF signals ($f < 30$ kHz) have been successfully employed over several decades for geophysical prospecting, submarine communications, and upper atmospheric remote sensing [1]–[5]. The primary reason is that ELF/VLF signals have the useful characteristic of being able to penetrate conductive barriers that are otherwise inaccessible for higher frequency radiation [6]. We explore the ability of ELF/VLF signals to penetrate metallic shells using a local transmitter driving an electrically small loop antenna. The motivation is to investigate the possibility of imaging through highly conductive media with a controllable near-field antenna. From a practical point of view, the analysis is relevant for nondestructive evaluation of conductive objects inside metal containers for industrial and security applications.

The general theoretical problem calculates the shielding effectiveness (SE) of a shielding structure via computing the fields before and after the presence of the shield. In general, the required shielding materials and geometry are strongly dependent on the frequency of operation, the type of source, and specific application [7]–[9]. Low-frequency electric fields, also known as quasi-electrostatic fields, can be shielded using a Faraday cage setup, which is simply a metal cage with apertures that are much smaller than the wavelength of the incident signal. Low-frequency magnetic fields, also known as quasi-magnetostatic fields, require either a mu-metal shield or high-conductivity material with extremely good electrical contact at all seals [10], [11]. At higher frequencies the electric and magnetic fields are coupled and a combination of the aforementioned techniques can be utilized to shield against electromagnetic radiation. For the purpose of this article, the effectiveness of highly conductive and nonmagnetic materials are considered for magnetic shielding when using a near-field loop antenna.

The physical mechanism of shielding quasi-static magnetic fields in conductors is via the flow of Eddy currents. The incident magnetic field will induce Eddy currents on the conductive shell that will in turn, by Lens’s law, be forced to cancel the incident magnetic flux. Thus, based on the geometry of the conductive shield, the permissible flow path of induced currents can dramatically alter the magnetic shielding effectiveness $S_E$ of a conductive enclosure [7], [12]. Additionally, since the location and size of the source relative to the shield directly influences the flow pattern of the Eddy currents, the type of source used can dramatically alter the shielding effectiveness [9], [12]–[14].

Over the past several decades, many research have investigated low-frequency shielding using analytical techniques [11], [15]–[17]. As with most analytical approaches, there exist only a handful of geometries for which the shielding effectiveness...
can be derived using closed form expressions [12]. Even so, many of the basic physical features of shielding emerge via analysis of basic geometries such as slabs, spheres, cylinders, and ellipsoids. In most of these analytically tractable scenarios, the problem focuses on a homogeneous magnetic field, which is effectively the same as an infinitely large solenoid surrounding the shield. Previous work has also considered the effect of near-field signals and have shown that the penetration due to fields from local sources can differ considerably from the uniform magnetic field case [9], [13], [18]. This phenomenon is primarily attributed to the low wave-impedance associated with near-field loop antennas or equivalently near-field magnetic dipoles [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [9]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17]. The incident signals are, thus, better impedance matched to the high conductivity (low impedance) shielding dipole [17].

Section II reviews analytical shielding theory for selected shield geometries and source parameters. Section III describes the locally corrected Nystrom (LCN) method and the configurations used in the numerical simulations. Finally, Section IV compares the analytical estimates and simulation results with experimental data in a typical urban environment. The results delineate the relative importance of shield geometry and source type for magnetic field penetration at ELF/VLF.

II. QUASI-STATIC SHIELDING THEORY

In general, determining the magnetic shielding effectiveness of an arbitrary shield geometry, source type, and source frequency requires a rigorous solution to Maxwell’s equations. Over the past several decades, researchers have employed various analytical tools to compute $SE_{H}$ in different geometries [7], [9], [11], [12], [25]. In this section, however, three canonical shielding problems are considered. The first is the classic transmission line method of Schelkunoff [26] where a plane wave is incident on a thin conductive slab of infinite extent. Although this method is not appropriate for quasi-static shielding, it serves as a stepping stone for including near-field effects. The second approach modifies the transmission line method by replacing the plane wave by a local magnetic dipole source to more accurately describe near-field shielding. The last scenario is that of a thin magnetic slab of infinite transverse extent and thickness $d$. Specifically, the shield extends between $0 < z < d$ and is of infinite extent in the $xy$ plane. The incident signal is assumed to be a plane wave propagating in the $z$-direction with magnetic field phasor amplitude given by $H_{0} = H_{0}e^{-jkz}y$ for $z < 0$, where $k_{0}$ is the free space wavenumber. The geometry of the problem is shown in Fig. 1. It is important to note that for measurements in a region $kz \ll 1$, the incident plane wave is effectively the same as a uniform field, that is $H_{0} \approx H_{0}y$. Even though the magnetic field is approximately constant, the plane wave carries net Poynting flux and is, thus, not applicable to the quasi-static shielding problem. Even so, the plane-wave model serves as a starting point before making corrections for the more practically relevant near-field source case.

The infinite slab model can be solved with standard multilayer reflection and transmission coefficient calculation techniques [9], [26], [28]. Specifically, it is assumed that the incident plane wave propagates in a region with the free space wave impedance that is given by $Z_{w} = Z_{0} = 120\pi \Omega$. On the other hand, the high conductivity approximation of the wave impedance in the metal shield is given by $Z = (1 + j)\sqrt{\frac{\mu}{\sigma}}$ and the corresponding complex wavenumber is $k = \frac{1}{\mu_{0}Z}$, where $\delta = \sqrt{\frac{\mu_{0}}{\sigma}}$ is the skin depth in a highly conductive material. Since the wave impedance of each region is known, the shielding problem can be solved by calculating the transmission coefficient on the opposite side of the shield at $z = d$. The shielding effectiveness is given $SE_{H} = \frac{\delta}{\tau}$, where $\tau = \frac{H_{a}}{H_{0}}$ is the transmission coefficient. This technique was first applied to shielding effectiveness by Schelkunoff [26] and is sometimes referred to as Schelkunoff’s transmission line method or Schelkunoff’s circuit method. In correspondence with the original formalism, the transmission coefficient can be written as $\tau = RAB$, where the terms $R$, $A$, and $B$ correspond to one pass reflection, absorption, and multiple internal reflections, respectively. The closed form expressions for each term are given by

$$R = \frac{4Z_{w}Z}{(Z_{w} + Z)^{2}}$$

$$A = e^{-kd}$$

$$B = \frac{(Z_{w} + Z)^{2}}{(Z_{w} + Z)^{2} - (Z_{w} - Z)^{2}e^{-2kd}}.$$  

Combining all three terms and using the definition $SE_{H} = \frac{\delta}{\tau}$, the closed form expression for $SE_{H}$ is given by

$$SE_{H} = \cosh(kd) + \frac{1}{2} \left( \frac{Z_{w}}{Z} + \frac{Z}{Z_{w}} \right) \sinh(kd).$$

![Geometry of an infinite slab shield illuminated by a plane wave with a y-directed magnetic field. $H_{0}$ is the magnetic field in the absence of the shield.](image)
The hyperbolic terms in (4) reflect diffusion through the conductor. The defining feature of this model is that the shielding effectiveness accurately accounts for reflection loss, absorption loss, and multiple reflections within the conductor.

Although the transmission line method is useful at estimating shielding effectiveness for plane waves, the utility diminishes when considering a near-field source. The primary difference is that the wave impedance of a near-field source is different from that of a plane wave [9], [14], [28], thus, reflection losses due to impedance mismatches are considerably different when considering a near-field source.

Several authors have considered the shielding problem due to near-field sources [10], [29]–[31]. Even the simple case of an infinite slab shield results in an infinite series solution or integral expressions that require numerical evaluation [14], [28]. Although numerically computing the shielding effectiveness is a reasonable approach, closed form approximations provide quick insight into the impact of a near-field source.

Consider the geometry in Fig. 2, where a magnetic dipole with moment \( \mathbf{m} = -m\hat{y} \) is placed on the \( z \)-axis at \( z = -R \). The magnetic dipole is mathematically equivalent to a loop antenna with current \( I \) and area \( A \) with equivalent magnetic moment \( m = IA \). The magnetic field \( \mathbf{H}_0 \) evaluated along the \( z \)-axis in the absence of the shield is then given by

\[
\mathbf{H}_0 = \frac{m}{4\pi(z + R)^3}\hat{y}. \tag{5}
\]

Here, (5) represents the near-field magnetic fields only. Additionally, the near-field electric field can also be determined as

\[
\mathbf{E}_0 = -jk_0Z_0\frac{m}{4\pi(z + R)^2}\hat{x}. \tag{6}
\]

In this current form, (4) cannot be utilized since the incident wave is spatially varying quite differently than a low-frequency plane wave. However, since near-field signals carry no radiated power, these fields can be treated as evanescent waves [14]. If the fields in (5) are locally approximated by an evanescent wave at \( z = 0 \), the incident magnetic field is then given by

\[
\mathbf{H}_0 \approx H_{in}e^{-jk_{NF}z}\hat{y} \tag{7}
\]

where the term \( H_{in} = \frac{m}{4\pi R^2} \) corresponds to the magnetic field at \( z = 0 \) in the absence of the shield. The term \( k_{NF} \) is an effective imaginary wavenumber of the evanescent wave and is given by

\[
k_{NF} = k_0R. \tag{8}
\]

The expression in (8) shows that the impedance is proportional to the free space wavenumber (and, hence, frequency) and is, thus, small for wave frequencies that are low. In particular, \( k_0R \ll 1 \) is always true in the near-field region and, thus, \( Z_w \ll Z_0 \) for a near-field loop source. Since the conductive barrier typically has a low impedance as well, this allows considerably better impedance matching with the barrier, and hence, higher penetration and lower \( SE_{HF} \) than the plane wave counterpart. Additionally, the impedance is proportional to the distance from the source and, thus, increases when the magnetic dipole is moved away from the barrier. On the other hand, the shielding effectiveness is considerably lower than that of plane waves when the near-field source is in close proximity to the shield.

With this modified definition of the wave impedance, the shielding effectiveness from (4) can be used by substituting (8) for \( Z_w \). Theoretical analysis of the equivalent wave impedance of electrical and magnetic dipoles has been well studied and further discussion can be found at [17]. Specifically, the general expression for the wave impedance of a magnetic dipole that is oriented parallel to the shield is given by

\[
Z_w \approx Z_0\frac{-k_0R)^2 + jk_0R}{-(k_0R)^2 + jk_0R + 1}. \tag{9}
\]

The expression in (9) is valid for both the far and near-field regions and demonstrates the importance of source distance from the shield. It easily seen that (9) approaches (8) for \( k_0R \ll 1 \), which by definition is the near-field region. For \( k_0R \gg 1 \), the impedance of the dipole approaches \( Z_0 \) and the plane wave condition is recovered. The expression shown in (4) can be easily evaluated for quick shielding estimates and is, thus, often used in shielding practice [9], [10]. Although the transmission line model of shielding provides useful quantitative predictions for large shields, the infinite shield assumptions discards any important features of geometry. The following section considers the effect of a finite sized shield on magnetic field penetration.

B. Spherical Shield in Uniform Magnetic Field

The simplest analytically solvable shielding model with a finite sized shield is that of a conductive spherical shell of thickness \( d \) and outer radius \( a \). Specifically, the sphere is assumed to be immersed in a spatially uniform magnetic field \( \mathbf{H}_0 = H_0\hat{y} \). Treating the incident magnetic field as uniform in the quasi-static limit inherently suggests that the fields are produced in the interior of an infinitely long solenoid. This assumption imposes...
Fig. 3. Geometry of a spherical conductive shield in a uniform magnetic field. $H_0$ is the magnetic field in the absence of the shield.

a solenoid electric field that is rotationally symmetric around the $y$-axis. Equivalently, at low frequencies, this approximation is also valid for an incident plane wave with free-space wave number $k_0$ such that $k_0a \ll 1$. Fig. 3 shows the geometry of the spherical shield model.

The solution to the spherical shield problem in the quasi-static limit was first presented by King [11], the shielding effectiveness for a thin ($a \gg d$) spherical shield is given by

$$SE_H = \cosh(kd) + \frac{1}{3} \left( ka + \frac{k}{ka} \right) \sinh(kd) \quad (10)$$

The hyperbolic terms in (10) once again reflect diffusion through the conductor and have a very similar form to (4). For the case of the spherical shield, however, the effect of the finite size is captured by the $ka$ terms in (10). Specifically, if the quantity $Z_w$ is replaced by $\frac{3}{2\sigma a}$ in (4), the expression would be the same as (10). However, in practical scenarios at low frequencies $Z_w \gg \frac{3}{2\sigma a}$ and $Z_w \gg Z$, which makes the shielding effectiveness of a spherical shell considerably lower than that of an infinite slab when the input signal is treated as a plane wave (or uniform magnetic field).

Fig. 4 shows a comparison of $S_{EH}$ for the infinite slab using the plane wave and magnetic dipole sources. The shielding values for the infinite slab case are also compared alongside $SE_H$ for the spherical shell. Specifically, the parameters used are $a = 0.7563 \text{ m}$, $d = 2.7 \text{ mm}$, $\sigma = 3.53e7 \frac{2}{\text{m}}$, and $R = 1 \text{ m}$. These shielding parameters correspond to an aluminum box shielding experiment that will be discussed in Sections IV and III. As shown, shielding of a magnetic dipole is inefficient at low frequencies and eventually converges to the more effective plane wave shielding solution (4) only for frequencies approaching 1 MHz. As mentioned previously, this is because the magnetic dipole impedance is better matched to the high conductivity layer and allows for higher field penetration. For the chosen parameters, the spherical shield provides the lowest value of $SE_H$ and differs from the plane wave slab case by over 100 dB in the ELF/VLF band. This feature clearly delineates the importance of a finite sized shield geometry in evaluating $SE_H$.

Although, the spherical shield has an even lower value of $SE_H$ than the near-field dipole case ($\approx 10$ dB), this is only true for the chosen value of $R = 1 \text{ m}$. If the distance of the magnetic dipole from the shield is reduced, the corresponding value of $SE_H$ can be made arbitrarily smaller. Thus, in principle, allowing for both a near-field source and a finite sized shield may permit even higher field penetration. Although the shielding effectiveness of a spherical shield near a magnetic dipole can be solved semianalytically [32], the expression for $SE_H$ is determined in terms of a spherical harmonic expansion and inevitably warrants numerical evaluation. Additionally, including a realistic and finite sized near-field source as opposed to a point dipole further requires a computational approach.

The analytical approximations described in this section provide practical estimates of shielding effectiveness in various applications. However, such simple models are not sufficient for high fidelity imaging applications [33] or for modeling complex shield geometries. For this reason, advanced numerical solutions are required for solving the shielding problem. Even so, appropriate selection of the computational framework is vital to provide quick and accurate predictions of field penetration through conductive shields [34]. In the following section, a surface integral equation formalism along with a Nyström numerical integration method is utilized to model the shielding effectiveness of a metal enclosure using a realistic near-field source.

III. LOCALLY CORRECTED NYSTRÖM METHOD

The problem of low-frequency magnetic shielding has been considered by several authors using a variety of numerical methods. In general, computational methods utilize numerical approximations to Maxwell’s equations that have been cast in
either differential equation or integral equation form. A comprehensive review of all computational electromagnetic techniques is beyond the scope of this article; for the sake of brevity and pertinence only relevant frequency domain methods are reviewed.

The most commonly used numerical technique is the finite element method (FEM) [35]. FEM requires the full domain of interest to be subdivided into a finite number of cells, or elements, extended to an exterior boundary with an appropriate radiation boundary condition, most commonly relying on an approximate absorbing boundary or an exact FEM boundary element method (BEM) [17]. FEM is well suited for problems with inhomogeneous or cluttered media and has been heavily utilized for shielding calculations [17]. Several commercial packages have further improved the ease of use and have thereby popularized FEM [36]. Even so, FEM methods are not ideally suited for thin shell shielded enclosures with a low surface area to volume ratio since a large number of cells can be required in the free space region inside the shield. Additionally, high conductivity shields would require several cells or a large number of high-order basis functions to accurately capture the skin effect [37].

An alternative is to cast Maxwell’s equations as an integral equation (IE) formulation [38]. Electromagnetic integral equations can be either expressed as a surface integral equation (SIE) or a volume integral equation (VIE). IEs typically utilize equivalent surface or volume currents as unknown auxiliary quantities that can be used to determine the unknown fields [38]–[40]. Any material region within the simulation that is not free space is first subdivided into elements. The primary advantage of IEs is that any free-space region does not need to be explicitly meshed and is implicitly captured by the kernel within the integrand of the IE [17], [38]. For a VIE, the material volume itself is discretized. The main computational advantage of the SIE is that equivalent currents are only placed on the boundaries separating material regions. In the case of a thin-shelled enclosure, the inner and outer surfaces of the shield are the surfaces along which the equivalent currents flow. Fig. 5 contains a visual depiction of equivalent surface currents on the exterior and interior of the conductive shell. The quantities given by $M_{ex}$ and $J_{ex}$ correspond to the surface currents on the exterior surface of the shell. Similarly, $M_{in}$ and $J_{in}$ correspond to the currents on the interior surface of the shell. Thus, unlike VIE, finite element or finite difference methods [41], the interior volume of the conductive region (where the wavelength can be extremely short or the skin depth extremely small) does not explicitly require cells through the thickness. This makes the SIE formalism extremely advantageous over other methods when considering problems of high shielding effectiveness. As such, SIE methods have been heavily employed in the past for thin-shell shielding calculations [42].

The primary concern with the application of the SIE to low-frequency shielding analysis is that the formalism must be carefully selected to avoid low-frequency instability. It is well known that the electric field integral equation (EFIE) operator (also known as the $\mathcal{L}$-operator) is unstable at low frequencies where the wavelength becomes significantly longer than the mesh cell size. Another challenge for shielding problems, which utilize highly conductive shields is that typical SIE methods such as the Poggio-Miller-Chan-Harrington-Wu-Tsai (PMCHWT) method become unstable for very high contrast materials. Common strategies for overcoming low-frequency breakdown and high contrast material issues include Calderón-based stabilization methods [43], [44] and Helmholtz-decomposition methods [45], [46]. Such methods typically require the use of special basis functions such as the Buffa–Christiansen elements [47], require operator products, and frequency scaling. An alternative approach is to use a current-charge formulation, also referred to as an augmented formulation [48], [49], which can also mitigate the low-frequency breakdown problem. Recently, an augmented Müller formulation was introduced, which was shown to be stable and accurate down to zero frequency and also stable for very high contrast materials [24]. In fact, at zero frequency, the formulation exactly decouples into the magnetostatic and electrostatic formulations. The advantage of this method is that it does not require operator products or frequency scaling. It also does not require the use of special vector function spaces, and is, thus, more suitable to solution methods such as the locally corrected Nyström method [50]–[53].

In this article, the augmented Müller formulation posed in [24] is applied to the low-frequency shielding problem. The SIE is discretized via the LCN method [50]–[53]. The LCN method has a number of advantages over the more traditional method of moments, including a high-order discretization at a reduced cost compared to method of moments. Also, for large problems, the point-based LCN discretization can provide flexibility when constructing fast solution methods. [52], The following provides an overview of the LCN discretization of the augmented Müller formulation.

Consider an unbounded region $V_1$ with complex dielectric constant $\epsilon_1$ and permeability $\mu_1$. An electromagnetic signal with electric field $\mathbf{E}_1$ and magnetic field $\mathbf{H}_1$ is present in $V_1$ and is incident upon an object composed of a homogeneous material with complex dielectric constant $\epsilon_2$ and permeability $\mu_2$. The incident fields are assumed to be solutions to the unperturbed problem in the absence of the object. The object occupies the volume $V_2$ and is separated from $V_1$ by the surface $S$. It is also assumed that the volume $V_2$ contains no sources in the interior. The presence of the object will result in additional fields that...
are scattered back into $V_2$ as well as fields that penetrate the interior of $V_2$. The total electric and magnetic fields in $V_1$ are, thus, given by $E = E_1 + E_2$ and $H = H_1 + H_2$ where $E_1$ and $H_1$ are the scattered fields. The fields in $V_2$ are given by $E_2$ and $H_2$. Equivalent electric and magnetic surface currents $J = n \times E$ and $M = E \times n$ as well as equivalent surface charges $p_e = n \cdot eE$ and $p_m = n \cdot \mu H$ are introduced on the surface $S$. The equivalent surface currents and charges also satisfy the continuity relationships, $\nabla_s \cdot J = -j \omega p_e$ and $\nabla_s \cdot M = -j \omega p_m$.

The augmented-Müller surface integral equation formulation combines the interior and exterior problems to uniquely determine the unknown equivalent charges and currents such that the true boundary conditions at the interface are satisfied [24]. The formulation results in a set of four IE that include the tangential electric and magnetic field IE (TEFIE and TMFIE) as well as the normal electric and magnetic field IE (nEFIE and nMFIE) that are given by

\[
\mu_1 (r) \cdot H_1 (r) = \frac{\mu_1 + \mu_2}{2} t(r) \cdot (J \times n) - \int_S t(r) \cdot (\mu_1 \nabla G_1 (r, r_o) - \mu_2 \nabla G_2 (r, r_o)) \times J(r_o) dS_o + j \omega \int_S t(r) \cdot M(r_o) \left( \mu_1 \epsilon_1 G_1 (r, r_o) - \mu_2 \epsilon_2 G_2 (r, r_o) \right) dS_o + \int_S t(r) \cdot (\nabla G_1 (r, r_o) - \nabla G_2 (r, r_o)) \rho_m (r_o) dS_o - \int_S n(r) \cdot (\mu_1 \nabla G_1 (r, r_o) - \mu_2 \nabla G_2 (r, r_o)) \times J(r_o) dS_o + j \omega \int_S n(r) \cdot M(r_o) \left( \mu_1 \epsilon_1 G_1 (r, r_o) - \mu_2 \epsilon_2 G_2 (r, r_o) \right) dS_o + \int_S n(r) \cdot (\nabla G_1 (r, r_o) - \nabla G_2 (r, r_o)) \rho_m (r_o) dS_o \quad (11)
\]

\[
\mu_2 (r) \cdot H_2 (r) = \frac{\mu_1 + \mu_2}{2} t(r) \cdot (J \times n) - \int_S t(r) \cdot (\mu_1 \nabla G_1 (r, r_o) - \mu_2 \nabla G_2 (r, r_o)) \times J(r_o) dS_o + j \omega \int_S t(r) \cdot M(r_o) \left( \mu_1 \epsilon_1 G_1 (r, r_o) - \mu_2 \epsilon_2 G_2 (r, r_o) \right) dS_o + \int_S t(r) \cdot (\nabla G_1 (r, r_o) - \nabla G_2 (r, r_o)) \rho_m (r_o) dS_o - \int_S n(r) \cdot (\mu_1 \nabla G_1 (r, r_o) - \mu_2 \nabla G_2 (r, r_o)) \times J(r_o) dS_o + j \omega \int_S n(r) \cdot M(r_o) \left( \mu_1 \epsilon_1 G_1 (r, r_o) - \mu_2 \epsilon_2 G_2 (r, r_o) \right) dS_o + \int_S n(r) \cdot (\nabla G_1 (r, r_o) - \nabla G_2 (r, r_o)) \rho_m (r_o) dS_o \quad (12)
\]

The augmented-Müller SIEs. The quantity $n$ is the local unit normal vector on $S$ pointing from $V_2$ into $V_1$. In contrast, $t$ is a unit test vector that is locally tangential to the surface $S$. The quantities $G_1 (r, r_o) = \frac{e^{-j k_1 r \cdot r_o}}{|r - r_o|}$ and $G_2 (r, r_o) = \frac{e^{-j k_2 r \cdot r_o}}{|r - r_o|}$ correspond to the homogeneous-space scalar Green’s functions in $V_1$ and $V_2$ with complex wavenumbers $k_1$ and $k_2$, respectively. The augmented-Müller equations in (11)–(14) have several desirable properties. The equations are second-kind Fredholm IEs, and thus, have a bounded eigenspectrum. At low frequencies, the operators are stable. In fact, at zero frequency, the cross terms, weighted by $j \omega$ drop to zero, completely separating the magnetostatic and electrostatic formulations. It was also shown in [24], that the condition number is stable for very high contrast and high loss materials, and does not suffer from interior resonance problems.

The augmented-Müller equations in (11)–(14) are discretized using the locally corrected Nyström method, following the procedures detailed in [53]. To this end, the surface is assumed to be discretized with a high order, curvilinear, quadrilateral mesh. The equivalent surface currents are expressed as unitary vectors sampled at quadrature points on the surface defined by mixed-order Gauss-Legendre quadrature rules ($(p + 1) \times (p + 1)$-point rules) [53], [54]. For local corrections, mixed-order Legendre polynomial basis with order $p \times (p + 1)$ weighting surface unitary vectors are used as basis. The tangential test vectors used for the TEFIE and TMFIE are unitary vectors [53], [55] sampled at the quadrature points. The surface charge densities are sampled at quadrature points defined by $(p + 1) \times (p + 1)$-point Gauss–Legendre rules. For local corrections involving surface charges, polynomial complete $p$th order Legendre polynomials are used.

A. Magnetic Shielding Simulations

We consider the shielding effectiveness of an aluminum container in the near field of a transmitting loop antenna. The parameters used in the numerical study are consistent with that of the performed experiment described in Section IV. Specifically, we consider a cubic aluminum box shield with side length $L = 1.2$ m, and thickness $d = 2.7$ mm. The bottom base of the box lies on the $xy$ plane, where the center of the base is located at $x = 0$ m and the $y$-axis bisects the base of the box. The transmitting loop antenna has radius $a = 0.5$ m and the center of the loop is located at $x = -(D + \frac{a}{2}) = -1.5$ m and $y = 0$ m, where $D$ is the distance from the closest face (see Fig. 6). It is important to note that the diameter of the loop (1 m) is on the order of the length of the box (1.2 m) as well as the distance between the loop’s center and the closest box face (0.9 m). This is a scenario where analytical techniques are completely intractable and is, thus, a demonstration of the LCN code’s utility. The numerical (and experimental) setup is illustrated in Fig. 6.

The simulation uses a total of 1728 surface elements with a mean edge length of 10.1 cm. The shielding effectiveness of the loop antenna is computed using the LCN method by running the simulation without the box and only the loop antenna first to provide $H_o$. The fields inside the shield are then computed by including the conductive box in the simulation to get $H_s$ in
Fig. 6. Illustration of simulation and experimental setup of an aluminum box shield in the near-field of a loop antenna.

Fig. 7. Shielding effectiveness computed numerically using Nyström-SIE simulation.

Fig. 8. Photograph of the open aluminum box with internal and external receivers in an indoor location prior to the experiment.

Fig. 9. Closed aluminum in the indoor preexperiment testing location. The blue cable is shown exiting the box via the aperture on the seam at the bottom-right corner of the box.

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the interior of the box region. The value of $\text{SE}_{H}$ is then found by computing the ratio $\frac{H_s}{H_0}$. An example case of a numerically computed $\text{SE}_{H}$ with a frequency of $f = 4$ kHz is shown in Fig. 7.

As shown in Fig. 7, the shielding effectiveness is close to zero outside the box region ($|x| > 0.6$ m) and is shown by the red dots. The interior of the box is depicted by the blue dots in Fig. 7. As shown in figure, $\text{SE}_{H}(x)$ is between 39 and 46 dB for the aluminum box shield. Since the value of $\text{SE}_{H}$ is a function of position inside the box, the geometric mean value is considered for comparison with experiment. If this is replaced by the maximum, median, or minimum value, the answers can change by approximately $\pm 5$ dB.

Each simulation of the LCN model was run using six processor cores (maximum of 12 threads) in under one minute, which further illustrates the clear utility of using the LCN model for numerical evaluation of shielding effectiveness. In the following section, the results of the LCN model are compared to experimental results in a noisy urban environment in the frequency range $1$ kHz $< f < 10$ kHz.

IV. EXPERIMENT AND VALIDATION

In order to validate the results of the simulations, the shielding effectiveness of a cubic aluminum box was determined by placing a small loop receiver inside the box and large transmitting loop outside the box. The parameters of the experiment are the same as those described in Section III. The box was continuously welded on five sides while the sixth face was bolted on. Fig. 8 shows a photograph in the preexperiment indoor testing environment of the box and loop receivers with the bolted face removed. As shown both the external and internal receivers are small loop antennas. The receiving antenna system utilizes a sophisticated design and is based on the atmospheric weather electromagnetic system for observation modeling and education (AWESOME) VLF receiver system. The specifications of receiver hardware is described in detail in [56]. It is also worth noting that the copper tape on the seams of the bolted face was removed for the final outdoor experiment.

A small aperture $<0.5$ cm was included at one seam to allow for cable connectors from the small loop receiver inside the enclosure. Fig. 9 shows the closed aluminum in the indoor preexperiment testing location. The blue cable is shown exiting the box via the aperture on the seam at the bottom-right corner of the box. It is well known that apertures can allow low-frequency
signals to enter the shield via leakage of Eddy currents [17], [57]–[60]. To mitigate these effects, the transmitting loop antenna was placed on the face opposite to the bolted side. In reference to Fig. 6, the bolted face is located at $x = -\frac{L}{2}$.

The experiments were conducted during heavy snowfall on February 22, 2018 in downtown Denver, CO, USA. Fig. 10 shows an aerial view (via Google Earth) of the field where the experiment was conducted on the University of Colorado Denver campus (GPS coordinates 39.746166°N, −105.004746°E). A photograph of the experimental setup is shown in Fig. 11. The loop antenna is placed according to the geometry of Fig. 6, however, additional antennas were placed outside the box to determine the external distortions to the magnetic field (similar to Fig. 9). The effect on the external measurement antennas is not considered in this article and will be discussed in future work. It is also worth noting that the mutual inductive coupling between the various loop antennas are not considered as part of this analysis. Inductive coupling will be necessary to take into account if the antennas are in close proximity, however, each antenna was at least 0.5 m away from its nearest neighbor in the experiment.

The frequencies considered in this experiment are between 1 and 10 kHz since the thickness of the box is between one half to two skin depths at these frequencies and appreciable penetration is expected to occur. Since these frequencies are in the audible spectrum, the transmitting loop was driven by a standard audio amplifier (Planet Audio AC1200.2 Anarchy 1200 Watt, 2 Channel, 2/4 Ohm Stable Class A/B, Full Range). The transmitter utilized a very basic design and simply consists of 10 winds of copper wire to build a 1 m diameter loop source. The transmitting format consists of a repeating frequency-time ramps from 1 to 10 kHz that last 5 s each. The instantaneous transmitted signal frequency in kilohertz as a function of time is, thus, given by

$$f(t) = \frac{9}{5}(t - t_0) + 1.$$  \hfill (15)

The expression in (15) is valid for $t_0 < t < t_0 + 5$, and $t_0$ is the initial transmit time of the ramp. Representative spectrograms (short-time Fourier transform) of the incident and penetrating fields are shown in Fig. 12. As shown, the incident field has considerably high SNR and the penetrating field inside the box is weak but clearly visible on the spectrogram. The weaker CW signals at approximately 20, 25, and 40.75 kHz are from distant Navy VLF transmitters, while the CW signal at approximately 32 kHz is believed to be from an unidentified local source. Fortunately, these signals do not interfere with the frequency band of the local transmitter and can be effectively ignored. Additionally, background noise in the ELF/VLF band typically contains several high-amplitude impulsive signals due to lightning radiation, or “sferics” [61]. As seen on the spectrogram in Fig. 12, the experiment happened to coincide with lower sferic activity, which made analysis considerably simpler.
The frequency ramps (as shown in Fig. 12) were specifically utilized since a single instantaneous frequency is defined as a function of time. Thus, once a ramp is demodulated in the time domain, the time axis can be replaced by frequency via the relation in (15). It is worth noting that the experiment was conducted over several hours and hundreds of ramps were transmitted. However, the driving audio amplifier signal was stable over the course of the experiment and the transmit ramps were effectively identical within the fidelity of the receivers. As such, only the analysis of the frequency ramp shown in Fig. 12 has been presented in this article.

Fig. 13 shows the measured values of $|H_0|$ (black) and $|H_S|$ (blue) as a function of frequency after demodulating the frequency ramps. As shown in figure, $|H_0|$ varies from a minimum of 56 dB (relative to an arbitrary reference) at 1 kHz to a maximum of 61 dB at 4 kHz. Thus, the incident field varies at most 5 dB over the band of interest. Inside the shield, $|H_S|$ has maximum at 1 kHz with a corresponding value of 26 dB even though the incident field has a minimum at 1 kHz. This feature demonstrates the well-known rule that lower frequency signals suffer less loss when traversing a conductive barrier. It is worth noting that although the fields undergo on average 45 dB of attenuation, the signal inside the shield is still 10–20 dB above the background noise floor, which makes calculation of $SE_H$ relatively straightforward.

The ratio of the penetrating fields (with shield) to the incident fields (no shield), thus, determines the penetration loss due to the aluminum box. A comparison of $SE_H$ as a function of frequency from the LCN simulation, experiment, and the basic theoretical models from Section II is shown in Fig. 14. $SE_H$ computed via the LCN code shows close agreement with the experimental data and deviates at most 3 dB. The additional plots on Fig. 14 show the theoretical predictions from the slab shield models as well as the spherical shell model. The various cases shown in Fig. 14 capture the relative importance of the type of source as well as the geometry of the shield. The case of a finite sized shield in the presence of a loop source (magnetic dipole) have the lowest values of $SE_H$ across all frequencies. If the source is made uniform, as in the case of the spherical shell theory, shielding becomes more effective, and $SE_H$ increases. If instead the shell is replaced by an infinite slab shield while keeping the magnetic dipole source, the impact is even greater than changing the source from a loop to homogeneous field. Finally, if the source is changed to a plane wave and the shield is turned into an infinite slab, $SE_H$ increases by over 100 dB. Thus, a near-field source that is in the close proximity of a finite sized shield has the greatest penetration. Although the shielding effectiveness decreases in these scenarios, such a methodology permits through conductor imaging applications for defect detection, noninvasive security
inspections of conductive containers, through bunker communications, and many other applications with complex geometries. Given the efficiency of the LCN method for modeling the penetrating fields, combining imaging, or shielding methodologies with a Nyström-SIE approach can alleviate computational bottlenecks associated with volumetric-based simulation methods and shielding design optimization.

V. CONCLUSION

Magnetic field penetration into a conductive box in the presence of an ELF/VLF transmitting loop antenna is investigated using theory, simulation, and experiment. In particular, we consider the regime where the loop diameter, shield size, and distance between source and shield are all of the same order. Rigorous analytical solutions are intractable in such a scenario. Using analytical approximations, we reviewed the magnetic penetration effects of a near-field magnetic dipole as well as the impact of a finite sized conductive shield. The analytical models provide insight into ELF/VLF shielding, however, the general problem of the realistic loop antenna and shielding structure is evaluated using a numerical approach. The work employs a broadband high-order locally corrected Nyström solution to the augmented Müller surface integral formulation of Maxwell’s equations (LCN code). The LCN code is an alternative to the typical BEM method and is stable for low-frequency shielding calculations. The LCN code is shown to match experimental data of shielding effectiveness within 3 dB. In contrast, analytical approximations are shown to overestimate the shielding effectiveness of the conductive box by 10–20 dB. By comparing the simulations to analytical estimates, the results elucidate the relative importance of source type, and shielding geometry for near-field signals in the ELF/VLF range.

REFERENCES


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