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A Novel Method for Determining the Lower Bound of Antenna Efficiency Using a Reverberation Chamber.

Master’s project directed by Associate Professor Mark Golkowski.

The use of antennas is becoming more wide-spread as common devices, from cell phones to laptop computers to automobiles, are being introduced into wireless communication networks. The increased use of antennas and wireless technologies is revolutionizing communications, but also making antenna characterization an integral part of system design. Many parameters of the antenna can be easily measured (i.e. radiation pattern, reflection coefficient), while others are virtually unknown. The efficiency of an antenna has been one of the more elusive parameters. It can be estimated, and theoretically calculated but accurately measuring it has been a challenge. A method for determining the lower bound of an antenna’s efficiency using a common electromagnetic environment: a reverberation chamber, and a theoretical formulation that models the antenna as a two-port network are shown. The proposed method is validated with experimental measurements. It is shown that further simulation work can allow for full characterization of an antenna in the context of a two-port model.
To my wife Christine and parents Jeffrey and Teresa Coder
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CHAPTER I

INTRODUCTION

The use of antennas has been increasing at an extremely rapid rate ever since their inception in 1888. Today, antennas of all shapes and sizes are in use throughout our everyday world. From common applications such as cellular telephones, television transmission, and land-sea communications, to more recent applications such as multiple-input multiple-output (MIMO) networks, passive and active radio frequency identification (RFID) tags, or automobiles, antennas are key components of modern technology.

As new, innovative antennas are designed, they need to be evaluated to determine if they perform to the required specifications. The most common parameters evaluated are the radiation pattern and reflection characteristics (i.e. standing wave ratio (SWR)). These two parameters are enough to properly characterize antennas for many applications, where coarse knowledge of the basic transmitting and receiving properties is sufficient. However, as transmitting and receiving systems become more sophisticated, antennas are being used in new ways, and must be modeled to greater precision. One example is the use of antennas to characterize the environment they operate in. This is being done in MIMO applications that utilize multiple antennas on both the transmitting and receiving ends. Using the channel characteristics, a communications system can alter its transmission frequency, bit rate, or modulation to a more optimal configuration.

In order for the channel characterization to provide accurate information, it becomes essential to know the parameters of the antenna(s) in use. If the antenna itself is included in the channel characterization, the system in effect measures the channel and the antenna. Not being able to separate the antenna parameters from the channel parameters can have adverse effects and cause the system to operate in a less than optimal configuration.
Work has begun on an antenna model and analytical and numerical methods that would allow for the separation of antenna parameters from channel parameters. To tackle this problem, a new model for antennas has been developed by Ladbury and Hill in [1]. Building upon this new model, analytical and numerical methods need to be developed that will allow for the full characterization of the antenna’s parameters. Full characterization is a long-term goal, and could take years to fully realize. This research is one step toward that goal. Determining antenna efficiency is a key component of the process.

Antenna efficiency is well defined from a theoretical perspective [2] but few practical measurements exist that can accurately yield the absolute efficiency [3]. There are several reliable methods for determining the relative efficiency of two antennas. Once an antenna is measured for absolute efficiency, relative methods could be used with increased accuracy. The method presented here shows one possible way to put a lower bound on the efficiency. A lower bound on efficiency is a relatively important advance because current methods yield highly variable results on the order of 65% efficient with an uncertainty of +/- 200% (for example). Being able to say the antenna in question can’t possibly be less efficient that a given value goes a long way to improving the design and characterization of antennas and the environments in which they operate.

**Scope and Organization**

This report is organized into seven chapters. Chapter I gives background knowledge and an introduction to the problem. To familiarize the reader with some of the uncommon aspects of this problem, Chapters II and III respectively discuss reverberation chambers and their applications and the current state of antenna efficiency measurements. This review isn’t intended to be a complete review of reverberation chamber principles and applications, rather a summary of the principles necessary to understand the research being discussed. Readers already fluent in
these areas of electromagnetics can skip these chapters. The conceptual two-port antenna model, along with its direct application to this problem, are discussed in Chapter IV. Chapter V presents the proposed method of determining the lower bound on antenna efficiency. Sections of Chapter V show the validation of the method via numerical simulations and observations. A numerical comparison to other methods is also presented. The current status of the long-term research project involving determining all of the antenna’s parameters is given in Chapter VI. Here, theory and results that have been reasonably validated are presented. Finally, the report finishes up with concluding remarks in Chapter VII.
CHAPTER II

OVERVIEW OF REVERBERATION CHAMBERS

Reverberation chambers are a relatively new innovation in electromagnetics and electromagnetic compatibility (EMC). Originally developed as a ‘screened room’ in the 1960s, reverberation chambers have grown into a viable test facility for a variety of applications that are traditionally done in anechoic chambers or open area test sites (OATS). Currently, reverberation chambers are being widely used to test products and model real-world environments.

A reverberation chamber can be thought of as the opposite of an ideal free-space environment (where there are no electromagnetic reflections). Constructed from metal panels, reverberation chambers work on the principle of reflections and scattering. Most reverberation chambers also contain a ‘paddle’ or ‘stirrer.’ This device is a large, metal object that moves within the chamber. The dynamic position of the stirrer changes the reflection geometry of the electromagnetic fields and allows for the creation of different electromagnetic environments. Figure 1 shows a typical reverberation chamber.
Figure 1. A typical reverberation chamber. The paddles are mounted floor to ceiling and wall-to-wall, each on a red shaft. The blue pyramidal object is foam designed to absorb RF energy.

Principles

One advantage of using a reverberation chamber is that from a statistical perspective, it creates a uniform electric field throughout the main chamber volume. This statistical uniformity is accomplished by ‘stirring’ the chamber. In a fundamental sense, stirring a chamber creates a new electromagnetic environment that contains different modes than the previous environment. A single chamber, as configured in Figure 1, can be stirred to create hundreds of statistically unique but equivalent electromagnetic environments. Averaging over the different environments yields a statistically uniform electric field.

This uniform field can be found throughout the “working volume” of the chamber. This volume is typically defined as one quarter of a wavelength from any metallic surface (i.e. walls, paddle(s)). Not included in this definition is the placement of other objects such as carbon-loaded foam. This foam is designed to absorb RF energy in the chamber. It has been shown that this or any absorbing material can locally degrade the field uniformity inside the chamber.
Using a reverberation chamber has a few disadvantages. First, because the energy transmitted into the chamber is thoroughly scattered, the radiation pattern of a device or antenna is not a factor within a reverberation chamber. The advantage of this is that there is no need to rotate a device when testing it for radiated susceptibility because the scattering ensures it is radiated from all possible incidence angles. Additionally, a reverberation chamber cannot be used at all frequencies. The lowest useable frequency (LUF) is generally defined in terms of chamber performance; the point at which the specified field uniformity can no longer be maintained. By contrast, the highest frequency at which a particular reverberation chamber can be used is not clearly defined. This limit comes from the physical design of the chamber and hardware limitations. At high frequencies, the losses (due to non-perfectly conducting walls) increase significantly, therefore requiring more power to maintain a given electric field level. The hardware used to construct the chamber may also become a factor. For example, bulkhead feed through connectors may have an upper limit of 18 GHz. Above this frequency, the connectors may produce unwanted reflections.

The statistics of the scattered electric fields inside a reverberation chamber are well known. The Cartesian fields (X, Y, or Z components) have a uniformly distributed phase (0 to $2\pi$) and Rayleigh distributed magnitudes. It is also known that the magnitude and phase of the electric fields are independent of each other. These distributions will be very important in the two-port model described in Chapter V.

**Applications**

Because reverberation chambers create a statistically uniform field, they can be used for investigating a variety of phenomena including radiated emissions, radiated susceptibility, material shielding, cavity shielding, and replicating an environment. Radiated emissions testing can be performed by putting the device under test (DUT) in the center of the chamber and
applying power to the DUT. While the DUT is powered up, the paddle can be discretely stepped or moved continuously (stirred) and the emissions from the DUT can be measured. Because the emitted energy is scattered into a statistically uniform field, an antenna – also inside the chamber – can be used to measure the field levels. These data can then be used to calculate the total radiated power from the DUT.

Radiated susceptibility works in an opposite manner; instead of the DUT emitting energy, an antenna transmits energy into the chamber as the paddles are turned. During this time, the DUT can be monitored for any adverse or undesirable events (i.e. unintended operation, device shutdown, sparks and/or flames).

Material and cavity shielding typically involve a technique referred to as ‘nesting’ reverberation chambers. A nested reverberation chamber configuration is where a smaller reverberation chamber is placed inside a larger one. Both chambers will have one or two antennas and typically (but not always) one or two paddles. Figure 2 shows a nested reverberation chamber setup that allows for the shielding of a material to be measured. For material and cavity shielding applications, energy is transmitted on an antenna in the large chamber and received on an antenna in the nested chamber, or vice versa.

![Nested Reverberation Chamber Setup](image)

Figure 2. Nested reverberation chamber setup for measuring material shielding.
The use of reverberation chambers to model real-world environments is a new application that is still being investigated. Researchers at the National Institute of Standards and Technology have been working to collect data from a multitude of buildings and outdoor environments (rural, urban and suburban) to try and isolate parameters of a communications channel that are unique to each environment [4]. For example, a high-rise office building has one set of channel parameters while a manufacturing facility might have a different set of parameters. Once these parameters have been identified, researchers hope to replicate each unique communications environment inside a reverberation chamber. The communications environment inside a reverberation chamber can be controlled two different ways.

First, carbon loaded foam – specifically designed to absorb RF energy – can be placed inside the chamber. This foam reduces the magnitude of reflections in the chamber. This type of control can replicate how reflective the real-world environment is. For example, a mineshaft with little metal will not be very reflective (and thus require a large amount of absorbing material) while an automotive manufacturing plant would require little absorbing material because the environment hosts a large number of reflections.

Second, the antennas can be oriented in such a way to increase or decrease the amount of line-of-sight coupling between them. The antennas can be moved physically closer or farther apart, or their polarizations can be changed (i.e. co-polarized or cross-polarized). Thinking of the previous example, the mineshaft would have a strong line of sight component, while the automotive manufacturing plant would have a relatively weak line of sight component.

Being able to re-create a real-world environment inside a reverberation chamber allows a wireless device manufacturer to test their product as if they were operating in the intended environment without having to physically travel to that particular environment.
CHAPTER III

TYPICAL EFFICIENCY MEASUREMENTS

There are several different methods of measuring the efficiency of an antenna. Most of these methods offer estimates of relative efficiency, or require the use of an antenna with a known efficiency. These methods can be less meaningful because relative efficiency is not very useful, and finding an antenna with a known efficiency is usually difficult. For the purposes of this overview, two popular methods will be discussed: the radiation pattern integration method and the Wheeler Cap method.

Definition of Efficiency

Before delving into the different methods of determining the efficiency of an antenna, efficiency itself must be formally defined. From a conceptual perspective, efficiency can be thought of as the ratio of power radiated from the antenna with respect to a known input power. Mathematically, this conceptual definition turns into the following equation [3]:

\[ E = \frac{P_R}{P_I} = \frac{P_R}{P_R + P_L}. \]  

(1)

In this equation, \( P_R \) represents the total radiated power, \( P_I \) is the total net input power and \( P_L \) is the total power lost. In (1), the left ratio is identical to the conceptual definition, while the ratio on the right is an equivalent expression that expands the input power into the sum of radiated power and lost power. This expansion can be done using the conservation of energy principal; all of the energy input into the antenna must be accounted for with energy either being radiated or lost.

Though deceptively simple from a mathematical perspective, the challenge comes in when any of the three power quantities are measured. Input power, \( P_I \), is the easiest to measure
because it is user controlled. Assuming the equipment used in the measurement is properly calibrated, measuring input power is relatively trivial. On the other hand, measuring the total radiated power and power lost are very difficult. In the next section, overviews of two popular methods for measuring the radiated power are presented. The discussion of the newly developed methods begins in Chapter IV.

**Radiation Pattern Integration Method**

The radiation pattern integration method is a straightforward method that involves sampling the electric field generated by the transmitting antenna under test (AUT) at points around the AUT. Mathematically, this process can be described as direct measurements of the quantity in the integral of the following expression [3]

$$P_R = \int_{S} P(\theta, \phi) \sin \theta \ d\theta \ d\phi .$$  \hspace{1cm} (2)

The surface of integration is a sphere that encloses the AUT. $P(\theta, \phi)$ is the radiation intensity over the surface of the sphere. With the AUT in the center of the sphere, the size of the sphere can vary. Ideally the radius should be such that the surface is in the far-field, but at low frequencies, this can be difficult. Contributing to this problem is the fact that the test facility (usually an anechoic chamber) needs to be sufficiently large enough to accommodate sampling around a sphere in the far field. Instrumentation required to collect this data can also contribute to the problems because supporting the instrumentation usually requires the use of metal support structures. Such structures cause electromagnetic reflections and impede the ability to take accurate data.

This method for determining efficiency can be very time consuming and yield high uncertainties due to the fact that as many points as possible – ideally an infinite number of points
must be sampled around the sphere to get a reasonable result. Also, this method is not very sensitive to small changes in antenna efficiency because the power is distributed across the surface of the sphere.

Wheeler Cap Method

First developed in 1959 by Harold Wheeler [5], the Wheeler Cap method has evolved into a popular method for measuring antenna efficiency. The key component of the Wheeler Cap method is a metal sphere (or hemisphere for an antenna working against a ground plane, i.e. monopole) that is placed around the AUT. Doing this allows one to measure some of the quantities in Equation (1). To use the Wheeler Cap method, the AUT should be viewed as a resistive network with ohmic losses. These ohmic losses can be due to the metal the antenna is constructed from and its radiation resistance. Viewing the AUT in this way, Equation (1) can be re-written to be:

\[
E_W = \frac{R_R}{R_R + R_L}
\]  

(3)

where \( R_R \) is the radiation resistance and \( R_L \) is the loss resistance [3]. The denominator of Equation (3) can be measured as the real part of the AUT’s input impedance. However, the real part of the input impedance only gives the quantity \( (R_R+R_L) \), not the amount each type of resistance contributes. The Wheeler Cap method gives a way to measure the contributions of each type of resistance.

The sphere that is used in Wheeler Cap measurements is to be one radian-length, or approximately 1/6th the wavelength [3] and have very low losses (highly reflective). A sphere of this size will act to short the antenna and prohibit it from radiating, allowing one to measure \( R_L \). One must be careful that the size of the sphere does not influence the current distribution on the
antenna. If it does, then the measurement of $R_L$ will not be accurate. On the other hand, if the sphere is too large, the antenna will generate modes in the sphere causing additional measurement inaccuracies.

Once the sphere is in place, a network analyzer can be used to measure the antenna without the sphere ($R_R + R_L$) and the antenna with the sphere in place ($R_L$). The disadvantage of this method is that the AUT should be less than one wavelength at the frequency of interest. This generally restricts this method to antennas that are not well matched. Measuring physically larger antennas, (i.e. log periodic) would be nearly impossible using this method.
CHAPTER IV

TWO-PORT ANTENNA MODEL

The two-port model used in this work was developed and first published by Ladbury and Hill [1]. This antenna model is novel, in that it models a real antenna as an ideal antenna (lossless and perfectly matched) connected to an unknown two-port network. All of the imperfections of the real antenna (mismatch and loss) are incorporated in the unknown network. Figure 3, from [1], depicts this setup.

![Diagram of Two-Port Antenna Model]

Figure 3. Antenna model proposed in [1].

The parameters of the two-port network model the efficiency and loss characteristics of the antenna. Thus knowledge of the two-port network parameters allows for full characterization of the antenna. It is important to note that the radiation pattern and scattering characteristics of the antenna(s) are not modeled as part of this two-port network, and are assumed to be identical to those of the real antenna. The unknown two-port model is defined such that the intrinsic impedance ($Z_0$) of port 2 is the same for port 1. Additionally, the network is defined to be reciprocal, that is, $S_{21}=S_{12}$ [1].

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By examining this model in a free-space environment, an equation for antenna efficiency can be derived. If a signal is transmitted into port 1 of the setup shown in Figure 3, there will be no reflections from our ideal antenna in free-space. Therefore, it will appear that the network is terminated using a non-reflecting load [1]. This results in the reflection coefficient of the antenna being the same as the $S_{11}$ of the two-port network. Given an incident power, $P_{inc}$, the power reflected from the two-port network is $P_{inc}|S_{11}|^2$, and the net power into the antenna (after the two-port network) is $P_{inc}(1-|S_{11}|^2)$. If the antenna after the two-port network is ideal, the net power into it – all of which will be transmitted – is $P_{inc}|S_{21}|^2$ [1]. Knowing this, the transmitting efficiency can be defined as:

$$
\eta_T = \frac{|S_{21}|^2}{1-|S_{11}|^2}.
$$

(4)

The receiving characteristics of this model can be expressed in a similar fashion. This model assumes that a non-reflecting load can represent the receiver connected to port 1. This results in a definition of receiving efficiency that is very similar to the transmitting efficiency equation:

$$
\eta_R = \frac{|S_{12}|^2}{1-|S_{22}|^2}.
$$

(5)

Though the receiving efficiency can be defined algebraically in a very similar way to transmitting efficiency, measuring $S_{22}$ of the model can be very difficult because it is still a vaguely defined quantity; it can’t be directly measured or calculated. It would be inappropriate to assume that $|S_{11}| = |S_{22}|$ because the two-port network used to model the antenna can be any reciprocal, realizable network. Not being able to assume the reflection coefficients are equal presents a problem: receiving and transmitting efficiencies may not be the same. For the most part, this issue will not
be addressed here. Later in this chapter, this issue is discussed as it relates to solving this problem in free-space. Chapter V contains a more in depth explanation of the differences between transmitting and receiving efficiencies.

In [1], Ladbury notes that the phase of $S_{21}$, $\phi_{21}$ is generally an unknown quantity and should have very little impact on the model. Note that the same is true for the phase of $S_{12}$, $\phi_{12}$, because it is assumed they are equal.

Historically, reverberation chambers have not been used for absolute efficiency measurements, and have been used to provide relative efficiency measurements. In this context, it is important to note that this work represents the first use of a reverberation chamber to determine the lower bound of absolute antenna efficiency.

**Free Space Model**

Though the model was originally defined for use with statistical distributions and reverberation chamber characteristics, this problem could *theoretically* be solved in a free space environment. The advantage of using a free-space approach is that the model becomes deterministic instead of statistical. However, accurately measuring the efficiency of an antenna in free-space can be very difficult. One of the measurements necessary to calculate efficiency is the total radiated power (given a known input power). In a free space environment, this means measuring power at a large number of points on a sphere around the antenna (known as the radiation pattern integration method). But, in a reverberation chamber, measuring the total radiated power of an antenna is very easy because the inherent scattering negates the radiation pattern of the antenna being measured. Regardless of the environment used to determine the final efficiency of the antenna, the efficiency result is valid in any environment. In other words, if the
efficiency of an antenna is determined using statistical methods in a reverberation chamber, the antenna will exhibit the same efficiency in a free-space environment.

Transmitting and Receiving Efficiency

It is important to note that though some discuss an antenna’s efficiency as though it is reciprocal - the same regardless of whether the antenna is transmitting or receiving – this is not strictly true. In reality, the transmitting efficiency of an antenna is not necessarily equal to its receiving efficiency. There are certain conditions where they will be equal; such as when \[ |S_{11}| = |S_{22}| \] (both are parameters of the two port model described above). However, unless one can be sure these conditions can be met, it shouldn’t be assumed the efficiency is a reciprocal term.

With this in mind, consider the prior discussion of using a statistical environment (reverberation chamber) compared to a deterministic environment (free-space). It has already been discussed what is involved in the calculation of transmitting efficiency, but what kind of measurements are involved in determining receiving efficiency in free-space? This is an ongoing part of this research, but the current thinking postulates that a plane wave incident on an antenna is partly received, and partly scattered by the antenna itself. How can the portion of the plane wave scattered by the antenna be measured? Since this measurement needs to be done for all incidence angles, the reverberation chamber seems like a good choice. However, the reverberation chamber environment may be too chaotic for distinguishing the incident energy from the energy scattered by the antenna.
CHAPTER V

LOWER BOUND OF ANTENNA EFFICIENCY

By expanding on the two-port model described in the previous chapter, an algebraic solution for transmitting and receiving efficiency can be derived. In order to develop these algebraic solutions, a careful assumption must be made. It will be assumed that, at least at a few paddle positions, the reverberation chamber is perfectly reflecting. That is, all energy that is transmitted into it is reflected back to the antenna. This assumption is reasonable as long as the frequencies being measured are above the LUF of the reverberation chamber, and the antenna is the dominant loss mechanism inside the chamber (as opposed to the walls or other contents). This assumption provides upper and lower frequency limits that depend on the specific reverberation chamber being used.

Figure 4 shows an expanded view of the two-port network being used to model the parameters of the antenna. When S-parameters are referred to in this section, it is in reference to the network shown here.

![Figure 4. An expanded view of the two-port network used to model an antenna.](image)

The $\Gamma_L$ parameter shown in Figure 4 represents the connection to the reverberation chamber using a perfectly matched and lossless antenna. With $\Gamma_L$ connected to the reverberation chamber, the statistical characteristics are known; uniformly distributed phase and a Rayleigh
distributed magnitude (technically it will be a \textit{truncated} Rayleigh distribution because there are a finite number of points). For a known \( \Gamma_L \), \( \Gamma_1 \) can be determined using the following equation from [6]:

\[
\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11} + \frac{S_{12}S_{21}}{1 - e^{-j\phi_L} - S_{22}}. \tag{6}
\]

In a reverberation chamber, the magnitude and phase of the electric field are independent of each other. This allows for the magnitude and phase of \( \Gamma_L \) to change separately, thus simplifying the analysis. If the magnitude of \( \Gamma_L \), \( r \) is held constant while the phase is varied, this would be equivalent to attaching a sliding termination to the model two-port network [1]. This concept has been well characterized, and it can be shown that equation (6) will transform a \( \Gamma_L \) circle (centered at the origin with radius \( r \)) to a new circle with the following radius and center [6]:

\[
R_1 = \frac{|S_{12}S_{21}\Gamma_L|}{1 - |S_{22}\Gamma_L|^2}, \tag{7a}
\]

\[
C_1 = S_{11} + R_1 |S_{22}\Gamma_L| e^{j(\phi_{12} + \phi_{21} - \phi_{22})}. \tag{7b}
\]

In [1], Ladbury and Hill discuss the impacts of the data transformation from \( \Gamma_L \) to \( \Gamma_1 \). This discussion is omitted here because when calculating efficiency, the mathematical manipulation of these equations is more important.

From an observational perspective, all of the parameters mentioned in equations (6) and (7), only one of them is known: \( \Gamma_1 \). To greatly simplify this analysis, it will be assumed that, at least for a few paddle positions, \( \Gamma_1 \) is equal to 1. To our advantage, these equations can be simplified further by remembering that the network is assumed to be reciprocal (\(|S_{12}| = |S_{21}|\)) and one can choose arbitrary phases for \( S_{21} \) and \( S_{12} \) (\( \phi_{21} \) and \( \phi_{12} \), respectively). In [1], it has also been
shown that because the statistical distributions of the magnitude and phase are known, $S_{11}$ can be found by taking the expected value of $\Gamma_1$:

$$E(\Gamma_1) = S_{11}.$$  \tag{8}$$

With these definitions and assumptions, the receiving and transmitting efficiency of the model two-port network can be found. To demonstrate the validity of the mathematical solutions, numerical simulations, a comparison to measured data will be shown. When using measured data, the $S_{11}$ values measured by a network analyzer will represent $\Gamma_1$ in the efficiency calculations.

**Receiving Efficiency**

When determining the receiving efficiency of the two-port network, it is crucial to remember the assumptions that were made at the beginning of this discussion. First, the reverberation chamber is perfectly reflective for some paddle positions; everything that is transmitted is reflected and there are no losses in the chamber other than the antenna. Because the reverberation chamber is perfectly reflecting, $|\Gamma_1| = 1$. Second, the network is reciprocal: $|S_{12}| = |S_{21}|$.

The receiving efficiency can be found by manipulating equation (7a) and utilizing the assumptions:

$$R = \frac{|S_{12}S_{21}\Gamma_L|}{1 - |S_{22}\Gamma_L|^2} = \frac{|S_{12}S_{21}|^2}{1 - |S_{22}|^2} = \frac{|S_{21}|^2}{1 - |S_{22}|^2} = \eta_R$$  \tag{9}$$

To calculate receiving efficiency, a set of $\Gamma_1$ data is taken and find the minimum radius circle that bounds all data points is found. According to equation (9), the radius of the minimum
Bounding circle is equal to the receiving efficiency of the antenna/two-port model network. This result has been confirmed in [6].

Though the algebraic solution for the receiving efficiency is simple, validating it with measurements will be difficult because incident energy to the antenna is partly received and partly scattered by the antenna. A method has yet to be found that will allow this quantity to be measured. Currently, the best that can be done is to compare receiving efficiency to transmitting efficiency. These two quantities may not be the same, but they should be similar.

**Transmitting Efficiency**

Finding the transmitting efficiency is not as easy as finding the receiving efficiency. In this process, the same assumptions that were used in finding the receiving efficiency will be used again to find the transmitting efficiency. Referring to Equation (4), both the transmitting efficiency and $|S_{22}|$ are unknown. To find both parameters, a simple system of equations can be setup. First, Equation (7b) can be used to find $|S_{22}|^2$:

$$ C = S_{11} + R |S_{22}| e^{-j\phi_{22}} \Rightarrow C - S_{11} = R S_{22}^\ast \Rightarrow S_{22}^\ast = \frac{C - S_{11}}{R} \tag{10} $$

The final result of Equation (10) can be solved to give $|S_{22}|^2$:

$$ |S_{22}|^2 = \frac{1}{R^2} |C - S_{11}|^2 \tag{11} $$

It should be noted that with a more detailed analysis of Equation (7b), the complex value of $S_{22}$ can be found. With $|S_{22}|^2$, Equations (4) and (5) can be manipulated to yield the transmitting efficiency:
Using Equations (11) and (12), transmitting efficiency can be determined using the same minimum radius bounding circle that was used to find the receiving efficiency.

Unlike receiving efficiency, transmitting efficiency can be compared to measured data. Current reverberation chamber methods provide relative efficiency between two antennas. For example, an AUT and horn measurement can be compared to a horn-to-horn measurement. These two measurements could yield the efficiency of the AUT compared to the horn antennas. However, the absolute efficiency of the horn antennas is generally unknown.

Numerical Simulations

The equations for transmitting and receiving efficiency were simulated using a script written in MATLAB. To numerically simulate these equations, a known $S$-parameter network was defined and associated with a randomly generated $\Gamma_L$ data set. This allows the script to calculate the efficiencies from data where the correct answer is known.

The numerical simulation script allows the user to select a transmitting efficiency and a value for $S_{11}$. The script then generates the other three $S$-parameters based on strict realizability conditions and the specified efficiency. Next, a set of $\Gamma_L$ data is generated at random. The user specifies the standard deviation and number of points. This random data is generated to have a uniform phase and a truncated Rayleigh distributed magnitude. The $\Gamma_L$ data is checked to ensure each point is within the unit circle. Any points outside the unit circle are thrown out and a new one is generated. With the fixed $S$-parameters and random $\Gamma_L$ data, $\Gamma_1$ is calculated. The minimum radius circle that bounds all $\Gamma_1$ points is then calculated. Figure 5 shows an example of randomly
generated $\Gamma_1$ data (left) and the same data transformed to $\Gamma_1$ (right). The red circle around the $\Gamma_1$ data is the minimum radius bounding circle. For this example, 1,000 random points were generated with a standard deviation of 1.25. This high standard deviation is necessary to obtain the truncated Rayleigh distribution needed to simulate low losses in the reverberation chamber.

![Figure 5. Left: Randomly generated $\Gamma_L$ data. Right: Same data transformed to $\Gamma_1$ with minimum radius bounding circle shown in red.](image)

The receiving efficiency is estimated directly from the radius of the bounding circle. The complex average of the $\Gamma_1$ data becomes the $S_{11}$ of the model network. The center of the bounding circle is used to find $S_{22}$ and the transmitting efficiency is calculated using equations (11) and (12). The transmitting and receiving efficiencies are calculated and compared to the values of the original S-parameter matrix.

The original assumption that $\Gamma_L = 1$ is maintained by a very high standard deviation (larger than 1). As the standard deviation decreases, the calculated values for transmitting and receiving efficiencies become significantly lower than the actual values. It is because of this departure from $\Gamma_1$ that these calculations remain a lower bound of efficiency. As $\Gamma_L$ decreases (to a limit of zero), the chamber is reflecting less energy and absorbing more energy.
Table I shows a list of standard deviations for randomly generated $\Gamma_l$ data sets. For all of these tests, the transmitting efficiency is set to 0.85 and $s_{11} = 0.0802 + 0.2891i$ (0.3 at 75 degrees). The fixed and calculated transmitting and receiving efficiencies are also shown. Notice how the calculated values diverge from the fixed values as the standard deviation is lowered.

**Table I. Sample Transmitting and Receiving Efficiencies.**

<table>
<thead>
<tr>
<th>Std. dev of $\Gamma_l$</th>
<th>RX efficiency (Fixed)</th>
<th>RX efficiency (Calculated)</th>
<th>TX efficiency (Fixed)</th>
<th>TX efficiency (Calculated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.8232</td>
<td>0.8221</td>
<td>0.85</td>
<td>0.8495</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8232</td>
<td>0.8214</td>
<td>0.85</td>
<td>0.8476</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8232</td>
<td>0.8181</td>
<td>0.85</td>
<td>0.8459</td>
</tr>
<tr>
<td>0.6</td>
<td>0.8232</td>
<td>0.8154</td>
<td>0.85</td>
<td>0.8434</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8232</td>
<td>0.8145</td>
<td>0.85</td>
<td>0.8413</td>
</tr>
<tr>
<td>0.3</td>
<td>0.8232</td>
<td>0.7945</td>
<td>0.85</td>
<td>0.8243</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8232</td>
<td>0.6249</td>
<td>0.85</td>
<td>0.6535</td>
</tr>
<tr>
<td>0.01</td>
<td>0.8232</td>
<td>0.0268</td>
<td>0.85</td>
<td>0.0286</td>
</tr>
</tbody>
</table>

The values in Table I show that the script is performing as expected. The mathematical core of this script is used again in the script that operates on measured data.

**Comparison to Measured Data**

As the final step in validating our algebraic methods for transmitting and receiving efficiencies, results for efficiencies calculated from this method are compared with relative efficiencies calculated from a reverberation chamber measurement. It is important to note that at this time, there is no alternative way to measure the receiving efficiency of an antenna in a reverberation chamber (or any environment).

The algebraic method shown in this chapter will be tested on two different antennas. Both antennas are meta-material inspired antennas, but one has a resonance near 300 MHz (well within
the useable frequency of the reverberation chamber being used) while the other has a resonance of 100 MHz (well-below the recommended frequency range for the reverberation chamber being used). Meta-material inspired antennas are on the cutting edge of antenna design. They are designed to be physically small (relative to a wavelength) and operate at low frequencies. These antennas are designed to be very narrowband and have a sharp resonance. A review of the theory and construction of meta-material inspired antennas can be found in [7]. Note that these are not strict meta-material antennas because they don’t contain any meta-materials. Rather, the design of these antennas was inspired by research in the meta-material field.

Each of these antennas was measured in a reverberation chamber with a reference antenna on one port and the meta-material inspired AUT. The measurement with the 100 MHz meta-material inspired antenna was done with a log-periodic antenna on port 2, while the 300 MHz antenna was measured with a dual-ridged horn on port 2. Because the only parameter necessary for these measurements is the reflection coefficient, there is only need for one antenna in the reverberation chamber. However, these measurements required two antennas because they are to serve as a direct comparison to relative efficiency measurements (which require two antennas). In fact, the second antenna impairs the proposed method due to the additional loss associated with the second antenna being in the chamber. But, the ability to do a direct comparison using a single set of measurements outweighed the problems caused by having a second antenna in the chamber.

Figures 6 and 7 show the first set of measured data. Figure 6 shows the calculated transmitting (TX) and receiving (RX) efficiencies for a log periodic antenna. Figure 7 shows the transmitting (TX) and receiving efficiencies for the 100 MHz meta-material inspired antenna.
Figure 6. Calculated receiving and transmitting efficiency of the log periodic antenna.

Figure 7. Calculated receiving and transmitting efficiency of the 100 MHz meta-material inspired antenna.
The transmitting and receiving efficiencies shown in Figure 6 are very similar. A few differences can be seen, but for the most part, the efficiencies are very close, except for the 80-100 MHz range. There are more striking differences between the transmitting and receiving efficiencies for the meta-material inspired antenna. Except for the resonances (which match up), the transmitting efficiency is fairly consistently near 50%, while the receiving efficiency is nearly zero. This is most likely due to a sampling and statistical distribution problem.

The genesis of this problem is that these measurements have a small number of $\Gamma_1$ points (300 for the 100 MHz measurements and 100 for the 300 MHz measurement). With so few $\Gamma_1$ points, the minimum radius bounding circle will be underestimated because there are very few outliers. As a result of this, receiving efficiency will be underestimated as well. Figure 8 shows an example of this.

![Figure 8. Sample plot from the meta-material inspired antenna showing an underestimated minimum bounding radius circle (left). A close-up view of the same circle is shown on the right.](image)

This problem only occurs when all of the following three conditions are met: low number of $\Gamma_1$ points, high mismatch, and high efficiency. This means that for the log periodic data and meta-material antenna (at resonance), our calculations are accurate. But, as Figures 7 and 8 demonstrate, when there is a high mismatch ($S_{11}$), the receive efficiency calculations are
underestimated. On the other hand, when efficiency is reasonable, and mismatch is low, the radius of the circle is very large (as one would expect) and receiving and transmitting efficiencies are more accurately calculated.

As an additional check, $|\Gamma_1|$, the reflection coefficient, can be examined. The resonances of the antennas should show up in $|\Gamma_1|$, in addition to on the efficiency plots. Figure 8 shows the $|\Gamma_1|$ plots for both antennas. It is expected that the $|\Gamma_1|$ plots look like the inverse of efficiency (approximately); where the reflection coefficient is low, efficiency should be high.

![Figure 9. Reflection coefficient values for the 100 MHz meta-material inspired antenna (top) and log periodic antenna (bottom).](image)

The strong resonances of the meta-material inspired antenna can be clearly seen in the reflection coefficient plot at about 100 MHz. The log periodic antenna also shows a characteristic resonance just after 100 MHz. Figure 8 provides a good indicator that the algebraic efficiency calculations are performing well.
Table II shows how these calculations compare to the relative efficiency method commonly used in the reverberation chamber. For this comparison, a frequency of 105.2 MHz is used. This is estimated to be the most resonant point of the meta-material inspired antenna.

<table>
<thead>
<tr>
<th>Frequency of interest</th>
<th>105.2 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated log periodic efficiency (proposed method)</td>
<td>80.9%</td>
</tr>
<tr>
<td>Calculated meta-material inspired efficiency (proposed method)</td>
<td>77.6%</td>
</tr>
<tr>
<td>Calculated difference between two antennas using the “relative method”</td>
<td>275%</td>
</tr>
<tr>
<td>Difference between two antennas using the proposed method</td>
<td>4.1%</td>
</tr>
</tbody>
</table>

Readers will notice that the difference calculated under the relative efficiency method is much higher than the difference calculated using the proposed method. This difference originates from something mentioned earlier in this report: large uncertainties. The relative efficiency method utilizes three measurements: reflection coefficient from each antenna and a through measurement from the log periodic antenna to the AUT. At the frequencies tested for this antenna, the field uniformity in the chamber is not very good, making measurement uncertainties for the through measurement very high. The proposed method does not utilize the through measurement, so its uncertainties are lower. However, the uncertainties for the proposed method are a little high because the calculation of the minimum bounding radius circle has not been optimized for a low number of points.

As another check of the proposed method, a 300 MHz meta-material inspired antenna was tested. Figure 9 shows the transmitting (TX) and receiving (RX) efficiencies for the dual-ridged horn antenna. Figure 10 shows the transmitting (TX) and receiving (RX) efficiencies for
the 300 MHz meta-material inspired antenna. Similar to the previous set of measurements, Figure 11 shows the \( \Gamma_1 \) (reflection coefficient) data for both antennas.

Figure 10. Receiving (top) and transmitting (bottom) efficiencies for the dual-ridged horn antenna.
Figure 11. Receiving (top) and transmitting (bottom) efficiencies for the 300 MHz meta-material inspired antenna.

Figure 12. Reflection coefficient values for the 300 MHz meta-material inspired antenna (top) and dual-ridged horn antenna (bottom).
As Figure 10 demonstrates, the dual-ridged horn antenna is designed to be a broadband antenna, exactly the opposite of the meta-material inspired antenna. Figure 10 shows the 300 MHz meta-material inspired antenna has a very strong and narrow resonance at approximately 307 MHz. As was seen in the previous test case, this meta-material inspired antenna shows an interesting characteristic in its transmitting efficiency; a noisy, but consistent 50% receiving efficiency at all frequencies outside of resonance. This is most likely due to the same reason as the 100 MHz antenna: a very high reflection coefficient and not enough samples to properly form the minimum radius bounding circle. The $|\Gamma_1|$ plots in Figure 11 corroborate the strong resonance seen in the meta-material inspired antenna.

Table III shows how these calculations compare to the relative efficiency method commonly used in the reverberation chamber. For this comparison, a frequency of 307.8 MHz will be used. This is estimated to be the most resonant point of the meta-material inspired antenna.

<table>
<thead>
<tr>
<th>Table III. 300 MHz Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of interest</td>
</tr>
<tr>
<td>Calculated horn efficiency (proposed method)</td>
</tr>
<tr>
<td>Calculated meta-material inspired efficiency (proposed method)</td>
</tr>
<tr>
<td>Calculated difference between two antennas using the “relative method”</td>
</tr>
<tr>
<td>Difference between two antennas using the proposed method</td>
</tr>
</tbody>
</table>

In this case, the two methods match up very well. At these, slightly higher frequencies, the reverberation chamber has much better field uniformity and the thru measurement utilized in the relative efficiency method does not have uncertainties as high as it does at lower frequencies.
Discussion

In both test cases, the reverberation chamber was operated at frequencies where it can be assumed that $\Gamma_L$ is close to one for some paddle positions, and that almost all of the energy transmitted into the chamber is reflected. As the values in Table I show, there can be some energy absorbed in the walls before there is a significant impact on the efficiency calculations. However, any departure from $\Gamma_L = 1$ will increase the measurement uncertainty.

One issue encountered in the comparison to relative efficiency data was that there are cases where the distribution of $\Gamma_L$ forces part of the minimum radius bounding circle outside the unit circle. This case typically occurs when the measured $\Gamma_L$ is very close to 1 (greater than 0.99). Figure 13 is an example of this problem, where the center of the minimum bounding circle is 0.2254-0.0844i and the radius is 0.8185. In this case, the calculated efficiency is erroneous because it is outside of the 0-100% bound.

![Figure 13. An example of a measured $\Gamma_L$ distribution that forces the minimum bounding circle to exceed the unit circle.](image)
Future research may show that this problem can be overcome, perhaps using an optimization method that forces the circle to be within the bound of the unit circle by weighting particular data points. In the results shown, these cases were simply filtered out of the final calculations. In the horn and log periodic calculations, these values were rare. However, in the meta-material data, these cases were prevalent as the reflection coefficient outside of resonance is almost always very close to 1. This problem would be alleviated by an increase in sampling. If the measurement were conducted with 1000 paddle positions, one would expect to see fewer cases where the radius of the minimum bounding circle exceeded the unit circle.

In comparison to the two methods discussed in Chapter III – the radiation pattern integration method and Wheeler Cap method – this method provides an easier method for determining the transmitting efficiency of an antenna. Neither of the two previously discussed methods provide any details on receiving efficiency. The use of a reverberation chamber eliminates the need for complicated spherical sampling to determine the total radiated power. Rather, this method uses the reflection of transmitted energy to calculate efficiency. This method also allows for measuring efficiency over a broad frequency range, unlike the Wheeler Cap, which is band limited by the size of sphere in use.

The disadvantage of this method is that it only provides a lower bound on efficiency. This is due to the fact the chamber does have some amount of loss at all paddle positions. As the number of points in the measurement increases, the lower bound of efficiency will become asymptotically closer to the true efficiency of the AUT.
CHAPTER VI
FUTURE WORK

Finding the transmitting and receiving efficiency of an antenna is only one part of this on-going research project. Additional efforts are being made to determine all eight parameters (magnitude and phase of each $S$-parameter) of the model network shown in Figure 4. The research done to find the efficiency resulted in the calculation of all but three parameters: phase of $S_{12}$ ($\phi_{12}$), phase of $S_{21}$ ($\phi_{21}$), and phase of $S_{22}$ ($\phi_{22}$). In [1], Ladbury and Hill stipulate that the phase of $S_{12}$ and $S_{21}$ are negligible because the problem is circularly symmetric. Part of this research effort was spent working on a method to determine $S_{22}$. The results shown here have been reasonably validated, but should not be considered hard fact. As this research progresses, the methods described here may be greatly improved and optimized.

This work is very similar to what was shown in previous chapters except that data with lower standard deviations are allowed, and chamber loss has a larger impact. Though $S_{22}$ was found in the previous sections, this is research deals with a larger range of possible data and makes fewer assumptions (i.e. it is no longer assumed that the chamber is perfectly reflecting at a few paddle positions).

**Extracting Antenna Parameters**

The determination of $S_{22}$ is very complicated because for any realizable network there are a range of possible values for $S_{22}$. The range of values can be calculated using the other three $S$-parameters, and is shown in [1] and [6]. The size of the range of possible values depends on the efficiency of the antenna. For a perfectly efficient antenna, there is only one possible value. But, as the antenna becomes less efficient, the number of possible $S_{22}$ values increases. As an extreme
case, if ports 1 and 2 are not connected at all, the efficiency will be zero, and $S_{22}$ can take on any value in the unit circle.

Though a range of $S_{22}$ values can be calculated for our model two-port network, there is no way of knowing which value will give the correct result. All values within the calculated circle are realizable, so another metric for sorting possible $S_{22}$ values must be found. Because this part of the research is in a very early stage, measured data has not been formally evaluated. The research has been based on statistical simulations, very similar to the code used to validate the proposed method for calculating efficiency.

This code begins by generating a random $\Gamma_L$ data set and assigning it a known, realizable $S$-parameter matrix based on a user-defined antenna efficiency and $S_{22}$. With this information, $\Gamma_1$ is calculated and from it, $S$-parameters are estimated. It is assumed that $S_{12}, S_{21}$ and $S_{11}$ are known because the network is reciprocal, efficiency of the antenna is defined, and $S_{11}$ can be found from the expected value of $\Gamma_1$. As with the code that simulated efficiency calculations, the correct answer is known. The goal is to get back to the known value through estimation and filtering.

Figure 14 (top left) shows an example of a random $\Gamma_L$ that is generated to have a uniform phase distribution and Rayleigh magnitude distribution. The top right panel shows the same random data transformed to $\Gamma_1$ using the set of user defined $S$-parameters, and a user-defined antenna efficiency of 60%. In this case, $S_{11}$ was set to $0.0802+0.2891i$, $S_{21}$ and $S_{12}$ were 0.7389, and $S_{22}$ was $0.1519+0.1734i$. In the lower left, the circle of all realizable $S_{22}$ values is shown. The red dot in the center represents the value chosen by the user. Ideally, the simulations and estimations will lead back to that value. The first analysis plot can be seen in the lower right. This plot shows the circular moving average of $|\Gamma_1|^2$. This plot shows whether or not the data is
“pulling” or biased toward one part of the complex plane. The current theory is that if the data is biased to a particular part of the complex plane, it can give some indication as to the value of $S_{22}$.

Figure 14. Example of random $\Gamma_L$ data (top left), corresponding $\Gamma_1$ data (top right), possible $S_{22}$ values (bottom left), and circular moving average of $|\Gamma_1|^2$ (bottom right).

Once the circle of $S_{22}$ values has been defined, it is gridded up and each value is checked to ensure that it forms a realizable network with the other parameters. Figure 15 shows the ‘sampled’ points. Points that are in green pass the realizability test, while points in red fail. The points around the edge of the circle fail due to rounding errors in MATLAB. This is not a concern because initial indications are that the true $S_{22}$ value is not on the edge of the circle.
At this point, the MATLAB script begins calculating $\Gamma_L$ for each realizable value of $S_{22}$. This calculated $\Gamma_L$ is then compared to the original, known $\Gamma_L$. Doing this allows different metrics and filtering methods to be tested to find the best value of $S_{22}$, that in turn gives the best $\Gamma_L$.

Exactly what metrics are to be used is still unknown. As a ‘check’ to ensure the script is working properly, the mean-squared difference between the real $\Gamma_L$ and the calculated $\Gamma_L$ (found by using ‘sampled’ $S_{22}$ points) is plotted. The top plot of Figure 16 shows the numerical difference vs. sample number. The bottom plot of Figure 16 shows the mean-squared difference in terms of intensity.

Figure 15. Sampled values from possible $S_{22}$ circle. Bottom shows a close-up view of the circle.
As expected, the $\Gamma_L$ sets calculated using ‘sampled’ values of $S_{22}$ that are very close to the real value show up as nearly zero mean-squared difference. As the samples move further away on the complex plane, the mean-squared difference gradually increases.

One strong metric that has been found thus far is the coefficient of variation of the calculated $\Gamma_L$ set. The coefficient of variation is defined as the standard deviation of $|\Gamma_L|^2$ divided by the mean of $|\Gamma_L|^2$. The coefficient of variation is a quick exponential distribution test. The random $\Gamma_L$ data has a coefficient of variance equal to 1 because it was generated from a known exponential distribution. This metric tests all calculated $\Gamma_L$ sets to see if they have the same
distribution as the original $\Gamma_L$ data. Figure 17 shows the coefficient of variation results based on sample number (top) and $S_{22}$ sample (bottom).

Figure 17. Coefficient of variation of calculated $\Gamma_L$ sets by sample number (top) and $S_{22}$ sample (bottom).

Another metric that has been investigated include the Kolmogorov-Smirnov (KS) statistical test. The KS test compares a test vector and sample vector to see what the probability is that the sample vector did not come from the same distribution as the test vector. In other words, it allows us to check if the phase values of $\Gamma_L$ are similar to a uniform distribution and magnitude values are similar to a Rayleigh distribution. The disadvantage of this metric is that any probability over 0.4 that the distributions are the same, are treated the same (i.e. a $p$ value of 0.4 is just as strong of an indicator as a $p$ value of 0.8). In these simulations, almost all of the generated
\( \Gamma_L \) sets had \( p \) values of 0.3 or higher, thus not giving any information as to which \( \Gamma_L \) set is closer to the original.

Unfortunately, there will most likely not be a single metric that yields the correct \( S_{22} \) value. The final answer will probably end up being a combination of metrics that yields a range of ‘good’ \( \Gamma_L \) sets. Then, the user can select one of these values and use it in the two-port model to further enhance their measurements. Once a solid set of metrics and thresholds have been defined, this script will be modified to accept and process measured data.

**Applications**

Once the two-port model parameters of an antenna can be defined, the uncertainty analysis of current electromagnetic measurements can be greatly improved. The ability to model the antenna as a complete two-port network will allow one to remove the effects of the antenna itself from the test environment, not just a reverberation chamber. This is a very powerful concept when two environments are being compared, or an environment is being replicated in a laboratory facility. This has a profound impact on adaptive MIMO sensor networks that ‘sense’ the communications channel and change their settings to transmit and receive information more efficiently.
CHAPTER VII

CONCLUSION

This report introduces a new method for determining the lower bound of absolute efficiency for an antenna placed inside a reverberation chamber. This method was derived algebraically based on equations from Ladbury and Hill [1] and waveguide theory [6]. Numerical simulations showed that the method produces good results with known data values, assuming key assumptions are met. Chiefly, the reverberation chamber must not be the dominant source of loss, and the antennas are reasonably well matched. A comparison with measured data and efficiency values calculated from another method yielded good results. These results are discussed and analyzed in light of two current methods for determining antenna efficiency.

Chapter VI details the current status of this research project and its efforts to determine all eight parameters (magnitude and phase of four S-parameters) for the proposed model. The most challenging of these parameters is $S_{22}$, which can only be estimated using statistical methods. Preliminary results show some possible methods for evaluating possible values of $S_{22}$ using a statistical distribution test.

Future efforts on this research project will be directed toward further evaluating this method of determining an antenna’s efficiency and finding the other three parameters of the two-port model proposed by Ladbury and Hill. Specifically, a formal uncertainty analysis should be done to determine how close to one $\Gamma_L$ must be in order for the calculation to be accurate. Some effort will also be devoted to developing a way to measure the receiving efficiency. This will allow for further validation of the proposed method.
REFERENCES


