Abstract—An open-loop current regulator for a high-speed synchronous reluctance machine with a solid conducting rotor is presented. A rotor dynamic model is developed which is similar to an induction machine model yet includes a magnetic saliency of the rotor. The model is then used to calculate command voltages for a desired current in an open-loop current regulator. Techniques for parameter extraction and discrete-time models for digital implementation are discussed. Experimental results, including a 120kW discharge of a flywheel energy storage system, validate the performance of the controller.

NOMENCLATURE

\[
\begin{align*}
\mathbf{L}_r &= \text{Rotor inductance matrix} \\
\mathbf{L}_s &= \text{Stator inductance matrix} \\
\mathbf{M} &= \text{Mutual inductance matrix} \\
\mathbf{R}_r &= \text{Rotor resistance matrix} \\
\mathbf{R}_s &= \text{Stator resistance} \\
\omega_{re} &= \text{Electrical rotor angular velocity} \\
\omega_r &= \text{Mechanical rotor angular velocity} \\
\mathbf{\lambda}_r &= \text{Stator flux linkage vector in rotor reference frame} \\
\mathbf{\lambda}_r^r &= \text{Rotor flux linkage vector in rotor reference frame} \\
\mathbf{i}_r &= \text{Stator current vector in rotor reference frame} \\
\mathbf{i}_s &= \text{Stator current vector in stationary reference frame} \\
\mathbf{i}_r^c &= \text{Stator current command vector in rotor reference frame} \\
\mathbf{v}_r &= \text{Rotor current vector in rotor reference frame} \\
\mathbf{v}_r^r &= \text{Stator voltage vector in rotor reference frame} \\
\mathbf{v}_s &= \text{Stator voltage command vector in rotor reference frame} \\
\mathbf{v}_c &= \text{Stator voltage command vector (dead-time compensated) in rotor reference frame} \\
\ell_s &= \text{Stator leakage inductance} \\
P &= \text{Pole number of the machine} \\
t_d &= \text{Dead or blanking time} \\
T_s &= \text{Switching period} \\
V_{bus} &= \text{DC bus voltage} \\
i_{spk} &= \text{Stator current magnitude} \\
\mathbf{J} &= \text{Rotation matrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}
\end{align*}
\]

I. INTRODUCTION

Current regulators for electric machines are typically based upon feedback control [1]. Implementation of field-oriented feedback control can be problematic for high-speed machines, however, as the dynamics of the machine in a field-oriented reference frame are speed-dependent. In particular, it can be shown that some imaginary component of the poles of the machine model in the rotor reference frame are essentially equal to the electrical rotor speed at high speeds. As a result, the system dynamics in the rotor reference frame become more resonant at high speeds, and hence it becomes more difficult to design stable feedback algorithms with good performance. Possible solutions to this involve gain-scheduling based upon rotor speed, but even so it can still be difficult to obtain good performance due to the inherent delays associated with a discrete-time implementation of the control algorithm, and also due to the presence of noise in the current measurements. Provided a sufficiently accurate model of the machine is available and is sufficiently robust to parameter variations, an open-loop controller can provide good performance and also avoids stability issues associated with feedback control, as the machine dynamics are inherently stable.

Synchronous reluctance machines have advantages in certain high-speed applications such as flywheel energy storage systems [3]. These machines have zero "spinning" losses when no torque is being generated by the machine, as opposed to permanent magnet machines with a stator iron. Furthermore, the rotor materials can be chosen to have good mechanical properties. The rotor of a synchronous reluctance machine design can possess excellent structural integrity if the rotor saliency is created by alternating layers of magnetic and nonmagnetic metals connected by a high-strength bonding process, such as brazing. However, under these conditions it is difficult to laminate the rotor, and therefore eddy currents can flow freely. As a result, the standard model for synchronous reluctance machines is inadequate, as it does not account for the resulting flux-linkage dynamics occurring in the rotor. In particular, when attempting a torque step from zero to full...
torque, the rate of change of the flux-linkage in the rotor is not instantaneous but determined by time constants. An open-loop controller based upon the standard synchronous reluctance machine model can therefore create a current overshoot during transients, as the predicted back-emf is much higher than the actual back-emf of the machine.

In this paper we present an open-loop controller for synchronous reluctance machines which takes into account the flux-linkage dynamics in a solid rotor configuration. First, the dynamic model of a solid-rotor synchronous reluctance machine is presented. Techniques for parameter extraction and discrete-time models for digital implementation are then discussed. An open-loop current regulator which uses this discrete-time model is then proposed. This current regulator is used in conjunction with a feedback voltage regulator to regulate the DC bus voltage of a flywheel energy storage system. Experimental results of such a system are presented and discussed.

II. FULL-ORDER MODEL OF SYNCHRONOUS RELUCTANCE MACHINE WITH ROTOR DYNAMICS

A. Continuous-time Model

Although a conducting rotor of a synchronous reluctance machine is technically a continuum system [2], it can be simply modeled in the rotor reference frame with direct and quadrature windings on the rotor, similar to the case of squirrel-cage induction machines. The equivalent two-phase flux-linkage/current relationships of the rotor and stator in the rotor reference frame are given by Faraday’s law:

\[
\begin{bmatrix}
\ddot{\lambda}_r \\
\ddot{\lambda}_q
\end{bmatrix} = \begin{bmatrix}
[L_s] & [M] \\
[M] & [L_r]
\end{bmatrix} \begin{bmatrix}
\dot{i}_r \\
\dot{i}_q
\end{bmatrix},
\]

(1)

where

\[
\ddot{x} = \begin{bmatrix}
x_d \\
x_q
\end{bmatrix}, \quad \dot{y} = \begin{bmatrix}
y_d \\
y_q
\end{bmatrix},
\]

(2)

the superscript 'r' represents the rotor reference frame and the 'd' and 'q' subscripts represent direct and quadrature values, respectively. The dynamic expressions for the machine in the rotor reference frame are provided by Faraday’s law:

\[
\frac{d}{dt} \ddot{X}_r = \ddot{\lambda}_r - R_s \dot{X}_r - J \omega_{re} \ddot{X}_r,
\]

(3)

\[
\frac{d}{dt} \ddot{X}_q = -[R_r] \dot{\lambda}_q,
\]

(4)

where \( \omega_{re} = \frac{P}{2} \omega_r \) is the electrical rotor speed.

By defining a new vector \( \ddot{X}_0 \):

\[
\ddot{X}_0 = \begin{bmatrix}
M \\
L_r
\end{bmatrix} \ddot{X}_r,
\]

we will choose the states of the system to be the vectors \( \ddot{\lambda}_0 \) and \( \ddot{X}_0 \). The dynamics can then be written as follows:

\[
\frac{d}{dt} \ddot{X}_r = - \begin{bmatrix}
[L_s] & [M] \\
[M] & [L_r]
\end{bmatrix} \ddot{X}_r + \begin{bmatrix}
R_s \\
R_r
\end{bmatrix} \dot{X}_r + \begin{bmatrix}
M \\
L_r
\end{bmatrix} \ddot{\lambda}_0 + \begin{bmatrix}
R_s \\
R_r
\end{bmatrix} \ddot{X}_0
\]

(6)

\[
\frac{d}{dt} \ddot{\lambda}_q = \begin{bmatrix}
L_s - M^2/L_r \\
L_r
\end{bmatrix} \ddot{\lambda}_q - \begin{bmatrix}
R_s + R_r M^2/L_r \\
R_r
\end{bmatrix} \dot{\lambda}_q
+ \begin{bmatrix}
L_s \\
L_r
\end{bmatrix} \ddot{X}_0 - J \omega_{re} \ddot{X}_0,
\]

(7)

where

\[
\ddot{X}_0 = \begin{bmatrix}
L_s - M^2/L_r \\
L_r
\end{bmatrix} \ddot{\lambda}_q + \ddot{X}_0.
\]

(8)

With this formulation the dynamics can be expressed in terms of three sets of direct and quadrature parameters: rotor time constants \( \frac{L_s}{R_s} \), rotor excitation resistance \( \frac{M^2}{L_r} \), and inductance \( \frac{L_s - M^2/L_r}{L_r} \), and a scalar parameter, the stator resistance \( R_s \).

B. Parameter Extraction

Both the direct and quadrature values of \( \frac{L_s - M^2/L_r}{L_r} \) are approximately equal to the stator leakage inductance, \( L_{ls} \), and hence can be estimated, as well as the stator resistance \( R_s \), through terminal measurements of the stator with the rotor removed. The parameters \( \frac{L_s}{R_s} \) and \( \frac{M^2}{L_r} \) can be determined from voltage and current measurements using the following procedure:

- Using a feedback current regulator at medium speeds, command either a direct or quadrature current \( \dot{i}_s \) to the machine, where the subscript 'x' stands for direct or quadrature values.
- Instantaneously turn off all transistors in the 3-phase inverter driving the machine at time \( t = 0 \). The stator current should quickly (ideally instantaneously) go to zero. In this case the stator voltage of the machine will be due solely to the flux generated by rotor currents:

\[
\ddot{v}_s = J \omega_{re} \begin{bmatrix}
M \\
L_r
\end{bmatrix} \ddot{X}_0 = J \omega_{re} \ddot{X}_0.
\]

(9)

From voltage measurements we can therefore easily determine the flux linkage \( \ddot{X}_0 \). From the flux waveforms we can also estimate the rotor time constants \( \frac{L_s}{R_s} \).

This can best be done through a curve fitting of the measured data. From the conditions at the turn-off transition \( t = 0 \) we can determine the rotor reaction resistances, for both direct and quadrature axes, as follows:

\[
R_{rx} \begin{bmatrix}
M^2 \\
L_{rx}
\end{bmatrix} = \frac{L_{rs}(t = 0^+)}{L_{rs}(t = 0^-)},
\]

(10)

The resulting parameters of a 120kW, 5500rpm solid-rotor synchronous reluctance machine are shown in Table I.
The natural dynamics of the machine when the speed of the machine is increased from 0 to 50000rpm. Arrows denote increasing rotor speed.

C. Operating Points [4]

The steady-state torque of a three-phase synchronous reluctance machine can be written in terms of its equivalent two-phase parameters as follows:

\[ \tau_{3ph} = \frac{3P}{4} (L_{sd} - L_{sq}) i_{sd}^r i_{sq}^r. \]  

For a given torque, there are a number of different combinations of \( i_{sd}^r \) and \( i_{sq}^r \) that can be used to achieve that torque. Typical operating points are as follows:

1) Minimum Current Operating Point: For a given torque, the peak current:

\[ i_{spk} = \sqrt{i_{sd}^r \tau_{3ph}^{2} + i_{sq}^r \tau_{3ph}^{2}} \]

(12)

can be minimized by choosing \( i_{sd}^r \) and \( i_{sq}^r \) to be equal:

\[ i_{sd}^r = i_{sq}^r = \sqrt{\frac{3P}{4} (L_{sd} - L_{sq}) \tau_{3ph}} \]

(13)

This is a desirable operating point if one wishes to minimize resistive losses in the machine, or if one is limited by the output current capability of the inverter.

2) Minimum Flux-Linkage Operating Point: Likewise, the peak flux linkage

\[ \lambda_{spk} = \sqrt{\lambda_{sd}^r \tau_{3ph}^{2} + \lambda_{sq}^r \tau_{3ph}^{2}} \]

(14)

can be minimized by choosing \( \lambda_{sd}^r \) and \( \lambda_{sq}^r \) to be equal. In terms of current, this results in the following relationship between \( i_{sd}^r \) and \( i_{sq}^r \):

\[ i_{sd}^r = \frac{L_{sq} i_{sq}^r \tau_{3ph}}{L_{sd}} = \sqrt{\frac{3P}{4} \left( \frac{L_{sq}}{L_{sd}} \left( L_{sd} - L_{sq} \right) \right)} \]

(15)

This operating point is desirable if one wishes to minimize core losses, which are proportional to the square of flux-linkage, or if one wishes to minimize flux levels to avoid saturation of the machine or to remain within voltage limitations of the inverter driving the machine.

3) Maximum Power Factor Operating Point: A third possible operating point is one which maximizes the displacement power factor, neglecting stator resistive drop. It can be shown that this operating point is achieved when

\[ i_{sd}^r = \sqrt{\frac{L_{sq} i_{sq}^r \tau_{3ph}}{L_{sd}}} = \sqrt{\frac{3P}{4} \left( \frac{L_{sq}}{L_{sd}} \left( L_{sd} - L_{sq} \right) \right)} \]

(16)

This operating point is desirable if one is limited by both voltage and current constraints, as it generates the most torque for a given voltage-current product. It is also a reasonably efficient operating point.

Other operating points than these can of course be chosen to achieve certain criteria, such as the maximization of efficiency [5].

III. CONTROL TECHNIQUE

A. Open Loop Controller

Because of the nature of a flywheel energy storage system (i.e., slowly changing rotor speed), it is straightforward to model the machine dynamics accurately. Hence, we can use the model developed above to determine the appropriate command voltages to be applied to the machine for a desired current. The voltages for a current \( \vec{\tau}_s \) and resulting air-gap flux \( \vec{\lambda}_a^r \) are given as follows:

\[ \vec{\tau}_s = R_s \vec{\tau}_s + \left[ R_e \left( \frac{M}{L_r} \right)^2 \right] \vec{\lambda}_a^r + J \omega_re \left( \frac{M}{L_r} \right) \frac{d\vec{\lambda}_a^r}{dt} + J \omega_r \vec{\lambda}_a^r + L_{el} \frac{d\vec{\tau}_s}{dt} \]

(17)

It will be shown that sufficient accuracy can be achieved by approximating the stator voltage as follows:

\[ \vec{\tau}_s \approx R_s \vec{\tau}_s + \omega_re \left[ \left( \frac{M}{L_r} \right) \frac{d\vec{\lambda}_a^r}{dt} + \frac{\vec{\lambda}_a^r}{L_{el}} \right] \]

(18)

The estimated vector \( \vec{\lambda}_a^r \) is determined from the desired stator current vector by numerically integrating the following
First-order techniques will be sufficient in this application. Hence a first-order technique (i.e., Forward Euler) is the timing is very tight when it comes to a high-performance drive. Therefore, we will need to compensate for the dead-time effect. The dead-time associated with a phase leg will alter the desired average-value output voltage of the phase as follows:

\[
<v_{out}(t)> = <v_{command}(t) > - \frac{V_{bus} t_d}{T_s} \frac{i_{out}(t)}{|i_{out}(t)|}.
\]

When calculating the command voltage in the rotor reference frame \(\vec{v}_{rc}\), we compensate for the fundamental component of the deadtime voltage using the desired current as follows [10]:

\[
\vec{v}_{rc}^* = \vec{v}_{rc} + \frac{4V_{bus} t_d}{\pi T_s} \frac{\gamma_r}{i_{spk}}.
\]

### D. Delay Compensation

Axis- and phase-transformations in the discrete-time domain cannot be ideally performed due to the continuous-time external system. The sample-and-hold effect in the stationary reference frame generates disturbances in the rotor reference frame, which can be seen in Fig. 4. This disturbance could be neglected when the switching vs. fundamental frequency ratio is large enough; however, this ratio cannot be sufficiently large in a high-speed system due to hardware limitations. Moreover, most hardware implementations require at least one sampling time delay to update the actual PWM command values. These delay factors make the estimated flux values and the actual voltage commands received by gate drive circuitry become significantly offset from expected values, as shown in Figs. 4 and 5. This can be solved by phase-shifting the angular velocity feedback information. By compensating the rotor position by \(\frac{3\omega_c T_s}{2}\), as shown in Fig. 8, the phase angle in the controller can be synchronized to the actual angle. This phase shift corresponds to the average phase for the next sampling period after the update delay of PWM commands. The compensated command voltages and the simulation results with compensated commands are shown in Figs. 6 and 7, respectively.
Fig. 3. Steady-state percent difference between the state variables of the first- and second-order approximated models when $i_{spk}$ varies from 0 to 1500A and the rotor speed ranges from 25000 to 50000rpm in typical operating modes. Upper row: Minimum current operating points, Middle row: Minimum Flux operating points, Lower row: Maximum power factor operating points. (a) Direct flux estimation (b) Quadrature flux estimation (c) Direct voltage command (d) Quadrature voltage command

![Graphs showing steady-state percent difference between state variables](image)

Fig. 4. Simulation: Disturbances in voltage commands by delay for the case of 15kHz sampling and 5000rpm rotation with $i_{sd} = 500$ A, $i_{sq} = -500$ A, $\nu^{r}_{sd}$, $\nu^{r}_{sq}$, $\nu^{s}_{sd}$, $\nu^{s}_{sq}$. Superscript 's' represents stationary reference frame. (a) Ideal (b) Actual (from top)

![Graphs showing voltage disturbance](image)

Fig. 5. Simulation: Erroneous flux estimation and current regulation by delay for the case of 15kHz sampling and 5000rpm rotation with $i_{sd} = 500$ A, $i_{sq} = -500$ A, $\lambda^{r}_{sd}$, $\lambda^{r}_{sq}$, $\lambda^{s}_{sd}$, $\lambda^{s}_{sq}$. (a) Estimated (b) Actual, $i_{sd}$, $i_{sq}$. (a) Command (b) Actual (from top)

![Graphs showing flux and current regulation](image)

E. Stationary Feedback Regulator

As the winding resistance of high-speed machines is quite low, asymmetries in the machine and applied voltage, though small, can generate significant low-frequency or DC currents in the stator. The purpose of the stationary regulator is to eliminate these currents by generating a feedback response voltage which cancels the low-frequency voltage component mentioned above. A block diagram of the entire current regulator implementation is shown in Fig. 8. Provided the fundamental electrical frequencies generated by the feedforward controller are much higher than the bandwidth of the stationary regulator, the stationary regulator achieves its purpose of eliminating the low frequency currents without interfering with the feedforward controller.
IV. EXPERIMENTAL VALIDATION

The proposed controller was validated on a 120kW, 4-pole synchronous reluctance machine. This machine is capable of speeds of up to 55,000 revolutions per minute. The rotor consists of alternating layers of a ferromagnetic and nonmagnetic material. This machine is part of a flywheel energy storage system manufactured by Pentadyne Power Corporation that is capable of providing 120kW of DC electrical power for up to 20 seconds. The system block diagram is shown in Fig. 9. The flywheel is suspended in vacuum by magnetic bearings. A picture of the machine rotor and flywheel rim is shown in Fig. 10. Fig. 11 shows the measured phase A current during a 500A step command using the feedforward control approach and a model without rotor dynamics, and the equivalent response of phases A and B using a controller where the rotor dynamics have been included. A clear overshoot of the measured current can be seen in the case where the rotor dynamics are neglected. The approximate rotor speed during these experiments was 39krpm. It can be seen that the model with rotor dynamics does not experience significant oscillatory behavior in the step response as seen in the simulations of Figs. 5 and 7. This fortunate result is most likely due to unmodeled damping mechanisms, such as core losses.

Fig. 6. Simulation: Phase-compensated voltage commands for the case of 15kHz sampling and 50000rpm rotation with $\tilde{v}_{rd} = 500 \text{ A}, \tilde{v}_{sq} = -500 \text{ A}$. $\tilde{v}_{rd}^e, \tilde{v}_{sq}^e, \tilde{v}_{rd}^p, \tilde{v}_{sq}^p$ (a) Ideal (b) Actual (from top)

Fig. 7. Simulation: Compensated flux estimation and current regulation for the case of 15kHz sampling and 50000rpm rotation with $\tilde{v}_{rd} = 500 \text{ A}, \tilde{v}_{sq} = -500 \text{ A}$. $\tilde{\lambda}_{rd}, \tilde{\lambda}_{sq}$, $\tilde{\lambda}_{rd}^e, \tilde{\lambda}_{sq}^e$, $\tilde{\lambda}_{rd}^p, \tilde{\lambda}_{sq}^p$ (a) Estimated (b)Actual, $\tilde{v}_{rd}^e, \tilde{v}_{sq}^e$ (a) Command (b) Actual (from top)

Fig. 8. Complete controller, including dead-time compensation, phase-delay compensation, and stationary regulator.

Fig. 9. Experimental setup of flywheel energy storage system

Fig. 10. 4-pole synchronous reluctance rotor and flywheel
current regulator is then used as an inner regulator in a bus voltage control algorithm, similar to that presented in [9]. The control logic initiates the regulation scheme when the DC bus voltage connected to the 3-phase inverter drops below 500V. Fig 12 presents the bus voltage and DC power supplied by the flywheel system when the DC power supply to the system is disconnected and a 120kW load is connected. The initial rotor speed during this experiment is 53krpm. It can be seen that the voltage regulator responds quite well to the application of an instantaneous load.

V. Conclusion

It is shown that a feedforward controller of a solid-rotor synchronous reluctance machine which includes the rotor dynamics is a valid technique for high-speed control. Provided the model parameters agree well with the actual system, good performance can be achieved. The most significant deviation between the system and the model is the saturation of the machine iron at high torque levels, which causes an effective reduction in the machine inductances, particularly the direct inductance. However, this problem can be resolved with a more sophisticated model of the flux-linkage/current relationships of the machine.

References