
VOLUME-BASED RUNOFF COEFFICIENTS FOR URBAN CATCHMENTS

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Abstract

The rational method is derived as a special case of kinematic wave flow. The definition of runoff coefficient is expanded from the flow-based ratio into volume-based ratio between the volumes under runoff hydrograph and rainfall hyetograph. The new method presented in this paper provides a dimensionless approach to theoretically derive the runoff coefficients for both the conventional two-flow drainage and low-impact cascading drainage systems. The key factors for determining runoff coefficients were identified as the ratios of depression loss and infiltration amount to the design rainfall depth. Although this method provides theoretical values for runoff coefficients, field data or computer simulation are always helpful for further validation.

Keywords

Runoff Coefficient, Kinematic Wave, Rational method

INTRODUCTION

This study presents an investigation to understand the definition of the runoff coefficient used in the rational method. In practice, runoff coefficients may be systematically derived using computer simulations and then verified with field data. For each observed rainfall-runoff event, both the time of concentration and the runoff coefficient are unknown. Therefore, the calibration process of runoff coefficient is not clear. In this study, the kinematic wave (KW) equations were integrated under the uniform rainfall distribution. The rational method is found to be a special case of KW flow, and the runoff coefficient can be derived using the volume ratio between the runoff hydrograph and rainfall hyetograph.

There are many numerical modeling techniques developed to convert a rainfall distribution into its runoff hydrograph through a catchment drainage network. Under a complicated condition, the dynamic wave (DW) approach is recommended, while the simple kinematic wave (KW) approach is also valid for cases without significant flow accelerations and backwater effects (Rossman 2005). Overland flows are often portrayed as a simple one-dimensional flow that is free from backwater effects under the approximation that the gravitation force is balanced with the friction force (Singh 1996). Using the KW approach, a natural catchment must be converted into its equivalent rectangle using the KW shape factor as (Guo and Urbonas 2009, Guo et al. 2012):

\[
\frac{L_w}{L} = (1.5 - \frac{A_m}{A})\left[\frac{2}{\pi} \sin\left(\frac{\pi}{8} \frac{A}{L^2}\right)\right] \quad \text{for } A/L^2 \leq 4
\] (1)
\[ X_w = \frac{A}{L_w} \]  

(2)

in which \( A = \) watershed area in \([L^2]\), \( B = \) watershed width in \([L]\), \( A_m = \) larger half area between left and right areas along the waterway through the catchment in \([L^2]\), \( L = \) length of waterway in \([L]\), \( X_w = \) length of overland flow in \([L]\), and \( L_w = \) width of KW plane in \([L]\) as illustrated in Figure 1.

**Fig 1. Conversion of Natural Catchment into KW Rectangular Plane**

On a rectangular plane, the KW governing equations for overland flow are well discussed in previous reports (Wooding 1965). The unit-width continuity and momentum equations are stated as

\[ \frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = I_e \]  

(3)

and

\[ S_f = S_o \]  

(4)

in which \( y = \) overland flow depth in \([L]\), \( q = \) unit-width flow rate in \([L^2/T]\), \( x = \) distance from the upstream boundary in \([L]\), \( I_e = \) excess rainfall intensity in \([L/T]\), \( S_f = \) friction slope, and \( S_o = \) surface slope. Eq (3) implies that the rating curve exists in a KW flow as:

\[ q = ay^\beta \]  

(5)

in which \( \beta = 3 \) for laminar flow or 5/3 to 3/2 for turbulent flow and \( \alpha = \) constant depending on surface slope and roughness coefficient (Wooding 1965). Taking the first derivative of Eq (5) with respect to time yields:

\[ \frac{\partial q}{\partial t} = U_e \frac{\partial y}{\partial t} \]  

(6)
$U_k = \alpha \beta y^{\beta - 1}$ \hspace{1cm} (7)

Where $U_k =$ KW speed in [L/T]. Substituting Eq (6) and (7) into Eq (3) results in

$$\frac{\partial q}{\partial t} + \alpha \beta y^{\beta - 1} \frac{\partial q}{\partial x} = \alpha \beta y^{\beta - 1} I_c$$ \hspace{1cm} (8)

Eq (8) can be integrated if the rainfall excess is steady and uniform. Applying the runoff coefficient to the uniform rainfall in Eq (8) yields:

$$I_c = CI$$ \hspace{1cm} (9)

Where $C =$ runoff coefficient. Although a uniform rainfall distribution reduces the intensity for peak flow prediction, it is still acceptable for volume studies because of the preservation of total rainfall depth over the event duration. Substituting Eq (9) into Eq (8), the total derivative of KW flow is derived as:

$$\frac{dq}{dt} = U_k CI$$ \hspace{1cm} (10)

Under a critical rainfall event in which the event’s duration is equal to the time of concentration of the catchment, Eq (10) is integrated over the period of time of concentration as:

$$q_p = \int_{q=0}^{q=q_p} dq = \int_{t=0}^{t=T_c} U_k CI dt$$ \hspace{1cm} (11)

Where $T_c =$ time of concentration in [T]. By definition, the time of concentration is the required travel time through the waterway or the overland flow length in this case.

$$X_w = U_k T_c$$ \hspace{1cm} (12)

Aided with Eq (12), Eq (11) is integrated to be:

$$q_p = U_k T_c CI = CIA$$ \hspace{1cm} (13)

Eq (13) is for a unit-width flow that can be further expanded to the entire catchment area as:

$$Q_p = q_p L_w = CIA$$ \hspace{1cm} (14)

where $q_p =$ unit-width peak flow in [L^2/T], and $Q_p =$ peak flow for the rectangular plane in [L^3/T]. When the rainfall is longer than the time of concentration, Eq (14) implies that the runoff coefficient is the ratio between the peak flow and its contributing rainfall intensity over the catchment area. For a longer rainfall event, Eq (11) can be integrated over the entire event. As a result, Eq (11) produces the volume ratio between the runoff hydrograph and the rainfall hyetograph as:
Where \( V_F = \text{runoff volume} \) and \( V_R = \text{rainfall volume} \). Eq (15) implies that the runoff coefficient is also equal to the ratio of runoff to rainfall volume through the entire event. Eq (15) is the definition of volume-based runoff coefficient that is independent of \( T_c \), while Eq (14) is the definition of flow-based runoff coefficient that needs the pre-knowledge of \( T_c \). In theory, Eq’s (14) and (15) shall agree to each other. In practice, Eq (15) is much easier to decode field data because it does not require \( T_c \). In this study, Eq (15) serves as the basis to derive runoff coefficients for engineering practices using the rational method.

**RUNOFF COEFFICIENTS**

For a small urban catchment, the land use can be divided into impervious and pervious areas. The watershed’s response to a rainfall event is very sensitive to how the storm drains are networked together. A distributed flow system in Fig 2 is laid with two separate flow paths to drain impervious and pervious areas respectively, while a cascading flow system is laid to spread stormwater from the upper impervious area onto the lower pervious area.

![Distributed Flow System](image1)

**Distributed Flow System**

Conventionally, roof areas are connected together through roof gutters that collect storm runoff from roofs and then drain onto the driveways. All driveways are linked through storm drains to pass stormwater directly to the adjacent streets. This drainage pattern is termed distributed system using two independent flow paths to drain stormwater from pervious and impervious areas respectively. A distributed flow system is efficient in stormwater removal, but it tends to result in higher peak and faster runoff flows. Considering that a catchment is composed of impervious and pervious areas, the rainfall volume for the given event is estimated as:

\[
V_F = CV_R
\]  
(15)
\[ V_R = PA \]  
(16)

Where \( P \) = precipitation depth in [L per watershed]. The runoff volumes produced from pervious and impervious areas are calculated as:

\[ V_m = (P - D_{vi}) I_a A \]  
(17)

\[ V_p = m(P - D_{vp} - F)(1 - I_a)A \quad m = 1 \text{ if } V_p > 0 \text{; otherwise } m = 0 \]  
(18)

\[ V_F = V_m + V_p \]  
(19)

Where \( V_m \) = runoff volume from impervious area in [L], \( D_{vi} \) = depression loss on impervious area in [L], \( I_a \) = impervious area ratio, \( V_p \) = runoff volume from pervious area in [L], \( D_{vp} \) = depression loss on pervious area in [L], \( F \) = infiltration amount in [L], and \( m = 1 \) if \( V_p > 0 \) or \( 0 \) if \( V_p \leq 0 \).

\[ C = \frac{V_F}{V_R} = n[1 - \frac{D_{vi}}{P}] I_a + m(1 - \frac{D_{vp}}{P} - \frac{F}{P})(1 - I_a)] \]  
(20)

\( n = 1 \text{ if } C > 0 \text{; otherwise } n = 0 \)

Like the rational method, the runoff coefficient in Eq (20) is linear with respect to watershed imperviousness. As a sum of two separated flows, Eq (20) is always dominated by the impervious areas or \( V_m \). Numerically, runoff coefficients in Eq (20) is always greater than zero as long as \( P > D_{vi} \).

**Cascading Flow System**

Under the latest development of green concepts for stormwater management, a lumped system represents a cascading flow system that drains storm water from impervious onto pervious areas. A lumped system consists of low-impact development (LID) devices or grass swales serving as a buffer zone to slow down runoff flows for filtering and infiltration purposes. In practice, the land uses within the project site do not produce a complete interception of the cascading flow. To model such a cascading flow, the catchment is divided into upper impervious and lower pervious areas (EPA SWMM). Mathematically, the interception of runoff volume generated from the upper impervious area is added to the lower pervious area as:

\[ V_p = m[r(P - D_{vi}) I_a A + (P - D_{vp} - F)(1 - I_a)A] \]  
(21)

Where \( r \) = flow interception ratio of \( V_m \). When \( r=1 \), Eq (21) represents a complete flow interception, while \( r=0 \), Eq (21) reproduces the flow condition in a distributed flow system. For \( 0<r<1 \), the residual runoff volume is directly released to the street as:

\[ V_m = (1 - r)(P - D_{vi}) I_a A \]  
(22)

The resultant runoff coefficient is calculated as:

\[ C = \frac{V_F}{V_R} = n(1 - r)(1 - \frac{D_{vi}}{P}) I_a + m[r(1 - \frac{D_{vi}}{P}) I_a + (1 - \frac{D_{vp}}{P} - \frac{F}{P})(1 - I_a)] \]  
(23)
Setting \( r=0 \), Eq (23) is reduced to Eq (20). Eq (23) produces smaller runoff coefficients than Eq (20) because of the additional infiltration losses. Numerically, Eq (23) can become zero if the catchment is under a low development condition. On the other hand, Eq (23) is converged to Eq (20) for a highly urbanized catchment because the lower pervious area has become too small to produce any more infiltration benefits.

**DESIGN EXAMPLES**

Both Eq’s (20) and (22) imply that runoff coefficients are related to the local design rainfall depth, and hydrologic loss parameters. Using the City of Denver as an example, the depression losses are recommended as listed in Table 1 (USWDCM 2001). The Horton’s formula is employed to describe the decay of soil infiltration rate as:

\[
f(t) = f_o + (f_i - f_o) e^{-kt}
\]

\[
F(t) = f_o + \frac{(f_i - f_o)}{k} \left(1 - e^{-kt}\right)
\]

Where \( f(t) \) = infiltration rate in [L] at time \( t \), \( f_i \) = initial rate in [L/T], \( f_o \) = final rate in [L/T], \( k \) = decay coefficient in [1/T], and \( F(t) \) = infiltrating amount in [L]. The recommended design values for infiltration and depression losses are summarized in Table 1 (UDSWMM 2001).

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Initial Infiltration ( f_i ) in/hr</th>
<th>Final Infiltration ( f_o ) in/hr</th>
<th>Decay Factor ( K ) in/hr</th>
<th>Pervious Depression ( D_v ) in</th>
<th>Impervious Depression ( D_i ) in</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.0</td>
<td>1.0</td>
<td>0.0007</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>B</td>
<td>4.5</td>
<td>0.6</td>
<td>0.0018</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>C/D</td>
<td>3.0</td>
<td>0.5</td>
<td>0.0018</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

*Table 1 Soil Infiltration and Depression Losses.*

Eq (24) describes the potential soil infiltration amount. In practice, the actual soil infiltration rate is the smaller one between the given rainfall intensity and Eq (23). In case that the uniform rainfall intensity is greater than the initial soil infiltration rate, Eq (24) is acceptable for providing the cumulative infiltration amount. Otherwise, the soil infiltration rate is varied between rainfall intensity and Eq (23) (Guo 1998). In this case, the calculation of rainfall excess was performed using the Colorado Unit Hydrograph Procedure (CUHP 2005). The CUHP is the calibrated hydrologic method developed for the metro Denver areas, the State of Colorado. It applies the unit-graph approach to predict runoff hydrographs from large watersheds. In order to achieve the regional hydrologic consistency in runoff predictions, the additional effort is to extend the CUHP’s application into catchments smaller than 90 acres. In essence, this task is to derive a set of runoff coefficients by which the rational method can agree with the CUHP for peak runoff predictions from small catchments. To derive the volume-based runoff coefficients, a large data base was established, including more than 250 urban catchments with their drainage areas varied from 0.03 to 1.0 sq mile. The computer model, CUHP 2005, was used to produce the runoff coefficients using Eq (13) under 1-hr storm event with recurrence intervals of 2-, 5-, 1-, 50-, and 100-yr. The infiltration amount was firstly determined and then weighted by the detailed computations for rainfall excess on pervious and impervious areas. As an example, all sample watersheds were defaulted to have a cover of Type C and D soils. Over the first one hour, the infiltration potential by Eq (24) was 3.0 inches, while the CUHP model...
reported the infiltration amount was 0.88 inch (22.4 mm). Using F=0.88 inch, the runoff coefficients are calculated for a distributed drainage system as summarized in Table 2.

<table>
<thead>
<tr>
<th>Recurrence Years</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>P in inches</td>
<td>0.95</td>
<td>1.35</td>
<td>1.60</td>
<td>2.20</td>
<td>2.60</td>
</tr>
<tr>
<td>Dv/P ratio</td>
<td>0.11</td>
<td>0.07</td>
<td>0.06</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Dv/P ratio</td>
<td>0.42</td>
<td>0.30</td>
<td>0.25</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>F/P ratio</td>
<td>0.93</td>
<td>0.65</td>
<td>0.55</td>
<td>0.40</td>
<td>0.34</td>
</tr>
<tr>
<td>Catchment Imp Lp</td>
<td>99</td>
<td>0.89</td>
<td>0.92</td>
<td>0.93</td>
<td>0.95</td>
</tr>
<tr>
<td>Runoff Coef C</td>
<td>85</td>
<td>0.76</td>
<td>0.79</td>
<td>0.83</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>0.58</td>
<td>0.62</td>
<td>0.68</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>0.40</td>
<td>0.45</td>
<td>0.53</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>0.22</td>
<td>0.27</td>
<td>0.38</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.04</td>
<td>0.10</td>
<td>0.24</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.00</td>
<td>0.05</td>
<td>0.20</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 2 Runoff Coefficients for Distributed Flow System

Fig 3 is the comparison between CUHP’s volume ratios derived using Eq (15) and the runoff coefficients determined using Eq (20). The minor differences are negligible for engineering practice. Fig 3 serves as the basis to have the rational method and the CUHP produce equivalent peak flows for small catchment hydrology studies.

Applying the same data set to Eq (23), another set of lumped runoff coefficients were also derived for the cascading or LID layout in this study. With r=1 or a 100% flow interception, the cascading flow process extends the additional infiltration losses to the 2-yr event only. For instance, as shown in Table 3, under \( I_p = 25\% \), the 2-yr event produces no runoff at all. Fig 4 presents the sensitivity of runoff coefficients to
flow interception ratio. As expected, the volume reduction effectiveness for cascading flow process on native C and D soils is limited to the 2-yr event under a catchment imperviousness<45%.

<table>
<thead>
<tr>
<th>Recurrence Years</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>P in inch</td>
<td>0.95</td>
<td>1.35</td>
<td>1.60</td>
<td>2.20</td>
<td>2.60</td>
</tr>
<tr>
<td>D_{v}/P ratio</td>
<td>0.11</td>
<td>0.07</td>
<td>0.06</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>D_{w}/P ratio</td>
<td>0.42</td>
<td>0.30</td>
<td>0.25</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>F/P ratio</td>
<td>0.93</td>
<td>0.65</td>
<td>0.55</td>
<td>0.40</td>
<td>0.34</td>
</tr>
<tr>
<td>Catchment Imp I_a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Runoff Coef C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>0.88</td>
<td>0.92</td>
<td>0.93</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>85</td>
<td>0.71</td>
<td>0.79</td>
<td>0.83</td>
<td>0.87</td>
<td>0.89</td>
</tr>
<tr>
<td>65</td>
<td>0.46</td>
<td>0.62</td>
<td>0.68</td>
<td>0.77</td>
<td>0.80</td>
</tr>
<tr>
<td>45</td>
<td>0.21</td>
<td>0.45</td>
<td>0.53</td>
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<tr>
<td>25</td>
<td>0.00</td>
<td>0.27</td>
<td>0.38</td>
<td>0.55</td>
<td>0.62</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
<td>0.10</td>
<td>0.24</td>
<td>0.45</td>
<td>0.53</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.05</td>
<td>0.20</td>
<td>0.42</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 3 Runoff Coefficients for Cascading (LID) Drainage System

Similarly, the aforementioned procedure can be repeated for Types A and B soils to produce their runoff coefficients. In practice, all runoff prediction methods are sensitive to the size of watersheds. When conducting a regional master drainage study, it is imperative that a set of runoff coefficients be derived to establish the basis of consistency between the rational method for small catchments and the unit-graph method for large watersheds. The new method derived in this study provides a guide as to how to conduct this needy task.
CONCLUSION

In the study, the rational method is found to be a special case of the KW approach. The runoff coefficient in the rational method is conventionally defined for the peak flow associated with the time of concentration of the contributing catchment. In this study, the integration of the entire storm event, the runoff coefficient is equal to the volume ratio between the runoff hydrograph and the rainfall hyetograph.

The new method developed for volume-based runoff coefficients is dimensionless. This approach provides theoretical values for runoff coefficients for both the conventional distributed two-flow drainage pattern and the cascading LID drainage pattern. A distributed flow system will always produce runoff flows as long as \( P > D_{10} \), while a cascading flow system promotes the green concept for more infiltration losses and results in lower runoff coefficients for the 2-yr event. How the difference becomes diminished when the catchment imperviousness is greater than 45%.

As a common practice, a regional drainage plan involves large and small catchments. It is always a challenge to establish the modeling consistency between unit-graph method for large watersheds and the rational method for small catchments. This new method derived in this paper can be very useful because it does not require the pre-knowledge of time of concentration. For the calibration of rational method, the soil infiltration amount becomes the best-fitted parameter between Eq’s (20) and field data.

REFERENCES


NOTATIONS:
A= watershed area,  
\(A_m\) = larger half area between left and right areas along the waterway through the catchment,  
B= watershed’s width  
C= runoff coefficient  
\(D_{vi}\) = depression loss on impervious area in [L],  
\(D_{vp}\) = depression loss on pervious area in [L],  
\(f(t)\) = infiltration rate in [L] at time t,  
\(f_i\) = initial rate in [L/T],  
\(f_o\) = final rate in [L/T],  
\(F\) = infiltration amount in [L],  
\(F(t)\) = infiltrating amount at time t in [L].  
\(I_a\) = impervious area ratio,  
\(I_e\) = excess rainfall intensity in [L/T],  
k = decay coefficient in [1/T],  
L = length of waterway,  
L_w = width of KW plane  
m = 1 if \(V_p > 0\) or 0 if \(V_p < 0\)  
r = flow interception ratio of \(V_m\)  
P = precipitation depth in [L per watershed].  
\(q_p\) = unit-width peak flow,  
\(q\) = unit-width flow rate in \([L^2/T]\),  
\(Q_p\) = peak flow for the rectangular plane  
\(V_m\) = runoff volume from impervious area in [L],  
\(V_{vp}\) = runoff volume from pervious area in [L],  
\(V_F\) = runoff volume  
\(V_R\) = rainfall volume  
\(S_f\) = friction slope,  
\(S_o\) = surface slope.  
\(T_c\) = time of concentration in [T].  
\(U_k\) = KW speed in [L/T].  
\(x\) = distance from the upstream boundary in [L],  
\(X_w\) = length of overland flow,  
\(y\) = overland flow depth in [L],  
\(\alpha\) = constant depending on surface slope and roughness coefficient  
\(\beta\) = 3 for laminar flow or \(5/3\) to \(3/2\) for turbulent flow